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Momentum and thermal slip effects on magnetohydrodynamic nanofluid flow with Brownian motion and thermophoresis effects

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Abstract:
This article aims on importance of thermal radiation on a viscous fluid flow over a uniform thickness stretching sheet. The mathematical model is formulated by using boundary layer theory and shooting mechanism has been utilised to solve the system of nonlinear differential equations. The momentum equation is considered with magnetic effect and energy equation has thermal radiation term. Velocity and thermal slip boundary condition are embedded in this study. It is observed that momentum profile decreases with slip parameter

Key words: MHD, Nanofluid flow, Shooting method.

1. Introduction.

Boundary-layer theory is used to model industrial manufacturing processes like extrusion, rolling, drawing, and coating of materials. In these processes, the motion of the surface creates a thin layer of fluid near the sheet, where important momentum and heat transfer occur. This idea helps researchers understand how velocity, temperature, and concentration fields change when fluids, especially nanofluids, flow over moving surfaces.(See [1], [2], [3]) The stretching sheet model is especially important in applications such as producing polymer sheets, cooling metal strips, drawing glass fibres, and cooling electronic surfaces. In these cases, controlling heat transfer rates is crucial for improving product quality and thermal efficiency. When the thickness of the sheet varies along its length, the model becomes more realistic and better reflects practical engineering systems with uneven shapes and changing thermal properties. Flow over a linear stretching surface is examined Crane [4].Kalidas [5] studied fluid flow over a permeable stretching sheet with Newtonian and partial slip boundary conditions.Bachok et al.[6] analysed boundary layer analysis of stagnation point flow over stretching and shrinking sheets. Hady et al.[7] reported the role of thermal radiation on stretching sheet viscous fluid flows. Khader et al.[8] studied natural convection effect on nonlinear velocity stretching sheet model. Khadir et al.[9] investigated non Newtonian viscoelastic rheological model past a stretching sheet. et al.[10] included the importance of buoyancy force term in viscous flows. An analytical solution is provided by Hassani [11] for magnetohydrodynamic nanofluid flow over moving surfaces. Mukhopadhyay and Gorla [12] investigated thermal radiation on a slippery continually moving surface. Similar works on Flow over stretching sheet can be seen in literature with different boundary conditions (See [13], [14], [15], [16], [17]).

The effect of thermal radiation is especially noticeable in situations such as high-temperature manufacturing processes, cooling of heated surfaces, energy systems, and biomedical heat transfer studies. Considering radiation in boundary layer analysis helps researchers better understand temperature behaviour and improve predictions of heat transfer performance. When radiation is present, the temperature inside the boundary layer usually increases faster compared to cases without radiation. As a result, the thickness of the thermal boundary layer may become larger, and the overall heat transfer rate from the surface changes. Because of this, radiation must be included in many heat transfer models to obtain accurate results. Mohamoud Ouaf [18] contributed exact analytical solution to MHD fluid flow over a stretching sheet with thermal radiation effect.Khan and makinde [19] studied radiation effect with respect to time dependent viscous fluid flows. Eugen Magyari et al.[20] reported on linearised Rosseland's radiation approximations. Anuar Ishak et al.[21] discussed the role of radiation in fluid flow over a vertical plate. Makinde et al.[22] studied on temperature dependent viscosity model on radiative flow over a plate. Motivated by the above studies, this present communication is aimed at investigating nanofluid flow over nonlinear stretching sheet with multiple slip effects. Many authors mentioned in the above cited research works mainly focused on stretching sheet with rough surface. But many industrial applications have slippery surfaces and this issue is not addressed by any of the investigators to the best of authors knowledge.

2. Mathematical modelling.

Let us consider a steady laminar incompressible fluid flow over a linear velocity stretching sheet which is moving with velocity $U_w(x) = bx^n$ where $b > 0$ is stretching constant. Flow configuration is explained in cartesian coordinate system where x- axis is in the direction of fluid flow and y-axis is perpendicular to the sheet. Figure-1 shows the physical representation of the model. Transverse magnetic field of uniform strength $B_0(x)$ is applied in the direction normal to surface and it is assumed that induced magnetic field has not significant due to low Reynold's flow configuration.

The boundary layer equations can be written as follows:

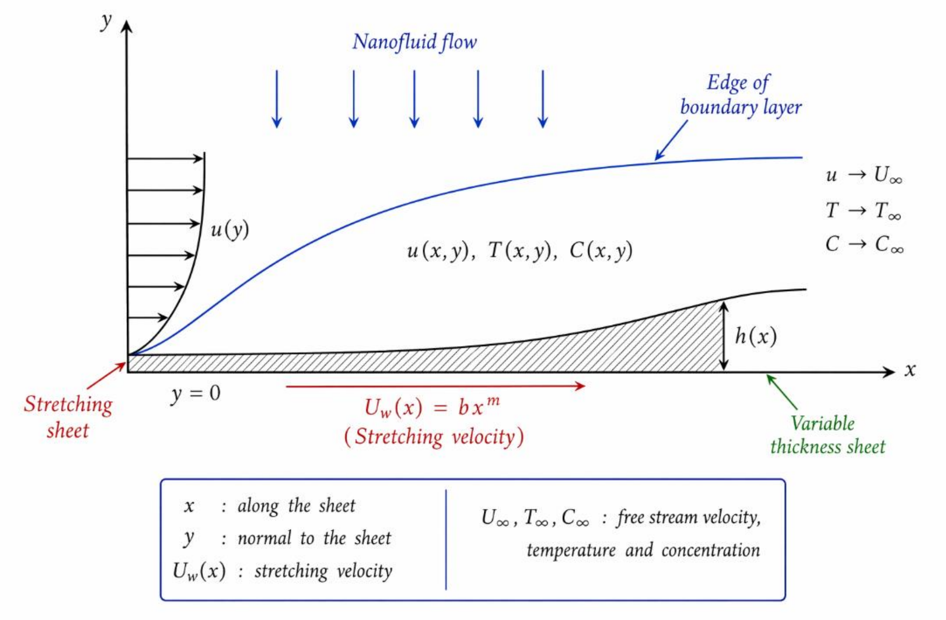

Figure 1: Sketch of the fluid flow configuration and conditions.
be expressed in Cartesian coordinates as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_f\frac{\partial^2 u}{\partial y^2}-{B_0}^2u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_f\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2\right]-(q_r)_y \tag{3}$$

$$\left\{u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right\}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The imposed boundary conditions on governing equations (1)-(4) are

$$u=U_w+\Delta_1\frac{\partial u}{\partial y},v=0,T=T_w+\Delta_2\frac{\partial T}{\partial y},C=C_w \quad \text{at } y=0,$$
$$u\to 0,T\to T_\infty,C\to C_\infty \text{ as } y\to\infty. \tag{5}$$

Here u, v indicates horizontal and vertical velocity components respectively, T being temperature and C is the nano particles volume fraction, β is Casson parameter, $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ here $(\rho c)_p$ is the notation used to indicate effective heat capacitance of the nanoparticles, $(\rho c)_f$ is the heat capacity of the base fluid, ν is the kinematic viscosity, D_B and D_T are Brownian, thermophoretic diffusion coefficients, k is permeability parameter. By using Roseland nonlinear thermal radiation approximation, the radiative heat flux q_r is written as $q_r=\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}=\frac{-16\sigma^*}{3k^*}T_\infty^3\frac{\partial T}{\partial y}$ where σ^*, k^* are Stefan boltzman constant, mean absorption coefficient respectively.
Introducing similarity transformations [23]

$$u=ax^nf'(\eta),v=-\sqrt{\frac{a\nu(n+1)}{2}}\,x^{\frac{n-1}{2}}\left(f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right),\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\emptyset(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{6}$$

The flow governing equation (2)-(4) reduces to the following form

$$f''+ff''-f'^2-Mf'=0 \tag{7}$$

$$\frac{\theta''}{Pr}+f\theta'+Nb\theta'\phi'+Nt\theta'^2=0 \tag{8}$$

$$\phi''+Lef\phi'+\frac{Nt}{Nb}\theta''=0 \tag{9}$$

Non-dimensional forms of Boundary conditions (5) are

$$f(0)=1+\delta f''(0),f'(0)=1,\theta(0)=1+\gamma\theta''(0),\phi(0)=1$$
$$f'(\eta)\to 0,\theta(\eta)\to 0,\phi(\eta)\to 0 \text{ as } \eta\to\infty \tag{10}$$

Here $Nt=\frac{\tau D_T(T_w-T_\infty)}{\alpha T_\infty}, Nb=\frac{\tau D_B C_\infty}{\alpha}, R=\frac{16\sigma^* {T_\infty}^3}{(\rho c_p)_{nf}3\alpha k^*}, M=\frac{\sigma {B_0}^2}{a\rho_f}$. where $Pr=\frac{\nu}{\alpha}$ is the Prandtl number, Nb is Brownian motion parameter, Nt is thermophoresis parameter, M is magnetic field parameter. Important engineering quantities namely skin friction co efficient and Nusselt number are defined as $c_f = \frac{\tau_w}{\rho_f u_w^2}$ and $Nu_x = \frac{x q_w}{k_f(T_w - T_\infty)}$ here τ_w ,q_w indicates wall shear stress, wall heat flux respectively.$\tau_w = \mu_f \left(\frac{\partial u}{\partial y}\right)_{y=0}$ and $q_w = k_f \left(\frac{\partial T}{\partial y}\right)_{y=0}$. In non dimensionalised form

$$C_f Re_x^{\frac{-1}{2}} = -\sqrt{\frac{n+1}{2}}\ f''(0) \quad \text{and} \quad N_u Re_x^{\frac{-1}{2}} = -(1+\tfrac{4R}{3})\sqrt{\frac{n+1}{2}}\ \theta'(0) \qquad (11)$$

3. Results and discussion.

The coupled nonlinear governing equations (7)–(9), representing conservation of momentum, energy, and nanoparticle concentration, are solved numerically using the shooting method in conjunction with an appropriate Runge–Kutta integration scheme after transforming the boundary-layer equations into similarity ordinary differential equations. The obtained numerical solutions are then used to analyze the influence of key dimensionless parameters on the velocity, temperature, and concentration distributions within the boundary layer. In particular, the effects of the magnetic field parameter M**,** Brownian motion parameter $\boldsymbol{N_b}$ thermophoresis parameter $N_t$are examined in detail.The magnetic parameter Mquantifies the relative strength of the externally applied transverse magnetic field and plays a significant role in controlling the momentum transport characteristics of the electrically conducting nanofluid. As illustrated in Figure 2, an increase in the magnetic parameter leads to a noticeable reduction in the velocity profile within the momentum boundary layer. Physically, this behavior arises due to the generation of a **Lorentz force**, which acts opposite to the direction of fluid motion and introduces a resistive drag force in the flow field. Consequently, the enhancement of magnetic field strength suppresses fluid motion and increases momentum boundary-layer resistance. This magnetic damping effect is particularly important in magnetohydrodynamic flow control applications, where regulation of fluid velocity is required for improved thermal performance and stability of transport processes

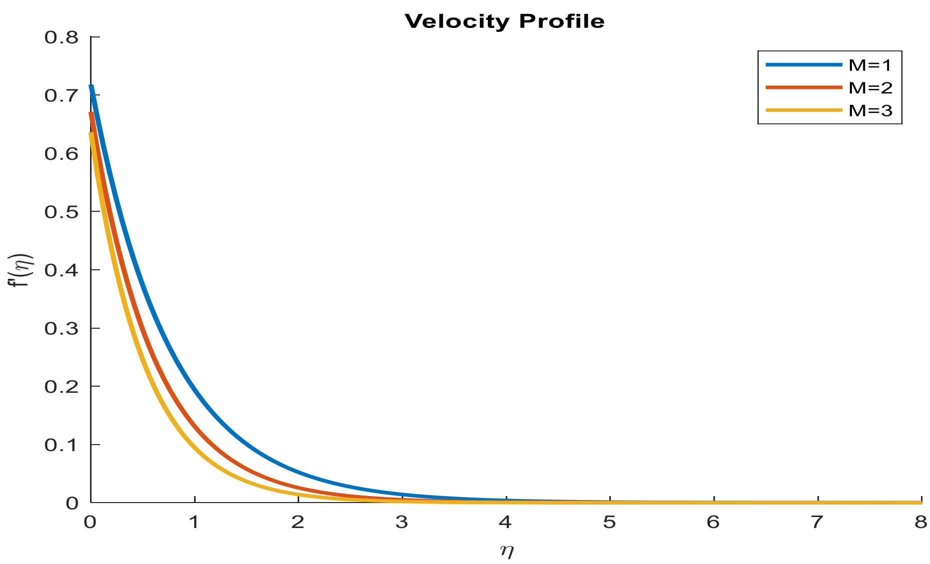

Figure 2: Effect of magnetic field on velocity profile.

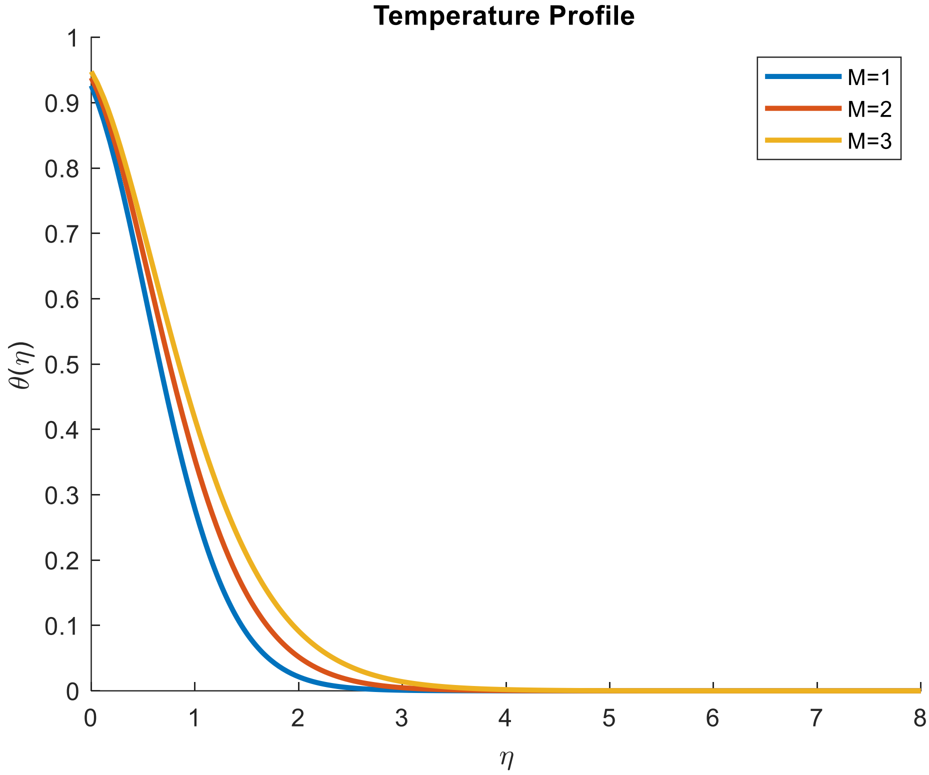

Figure 3: Effect of magnetic field on temperature profile.

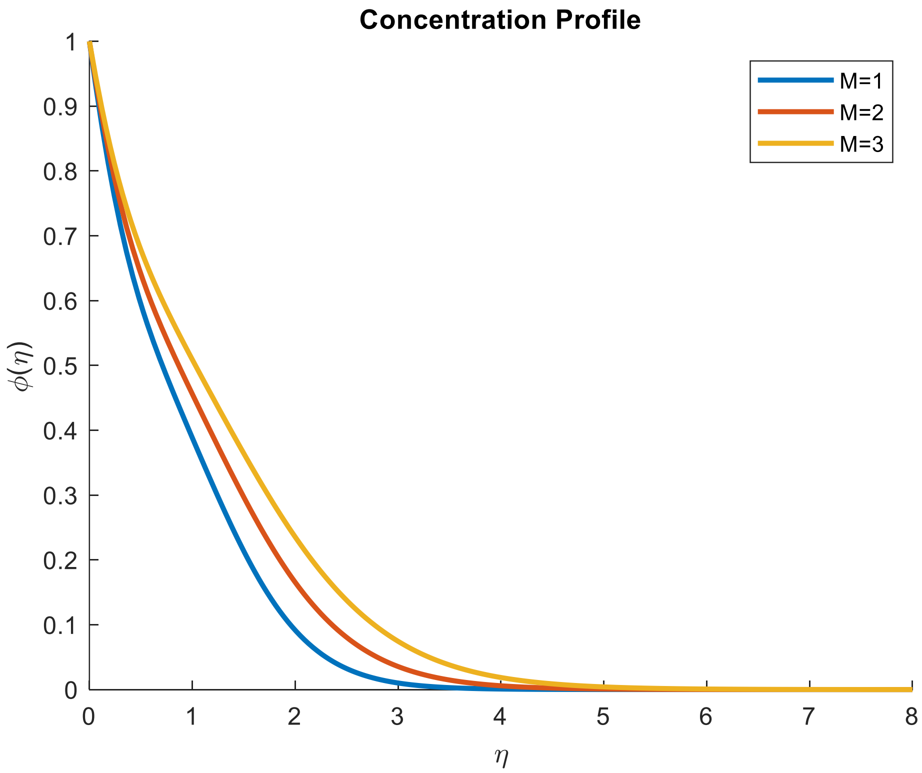

Figure 4: Effect of magnetic field on concentration profile.

Figure 3 shows the effect of magnetic parameter (M) on energy profile. Increasing the magnetic parameter increases the temperature profile because the applied magnetic field produces additional heat inside the fluid, which raises the fluid temperature. When the magnetic parameter increases, the temperature of the fluid increases. This happens because the applied magnetic field creates resistance to the fluid motion. Figure 4 shows the role of magnetic field strength on concentration profile. Increasing the magnetic parameter decreases the concentration profile because the applied magnetic field slows the fluid motion and reduces mass transfer within the boundary layer. When the magnetic parameter increases, the concentration usually decreases. This happens because the magnetic field slows down the fluid motion. As the flow becomes slower, the transport of species (such as nanoparticles or solute particles) away from the surface is reduced. Figure 5,6,7 shows the importance of slip parameter on velocity, temperature and concentration profile.

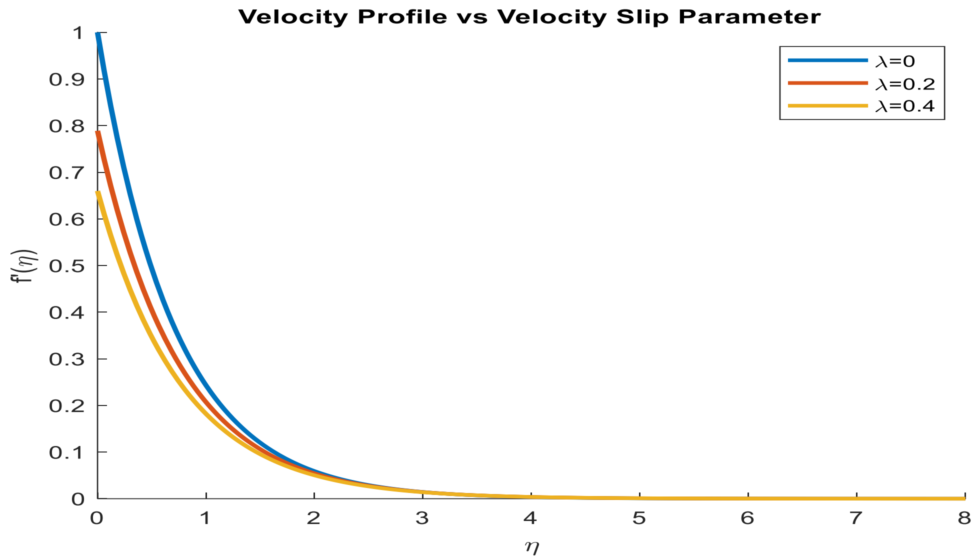

Figure 5: Effect of slip parameter on velocity profile.

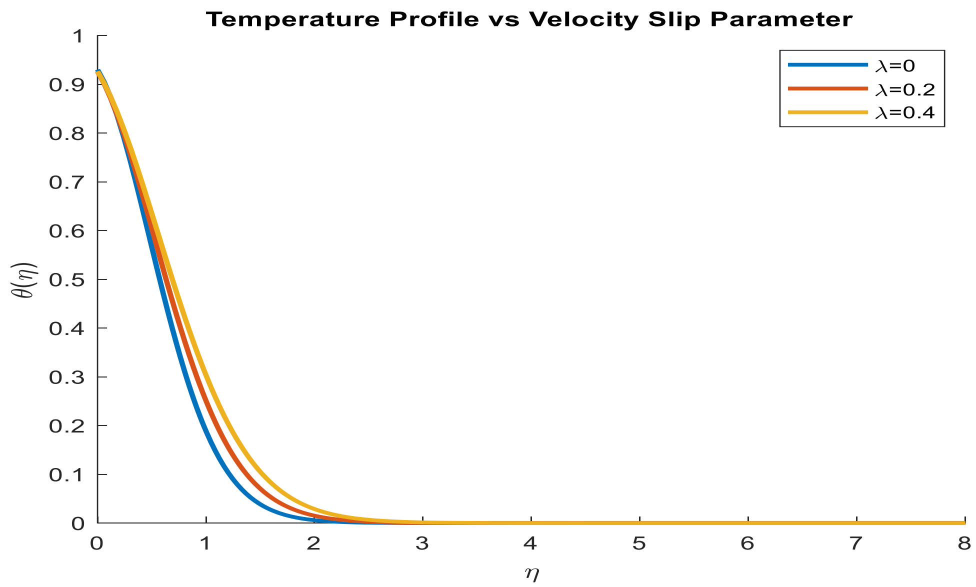

Figure 6: Effect of slip parameter on temperature profile.

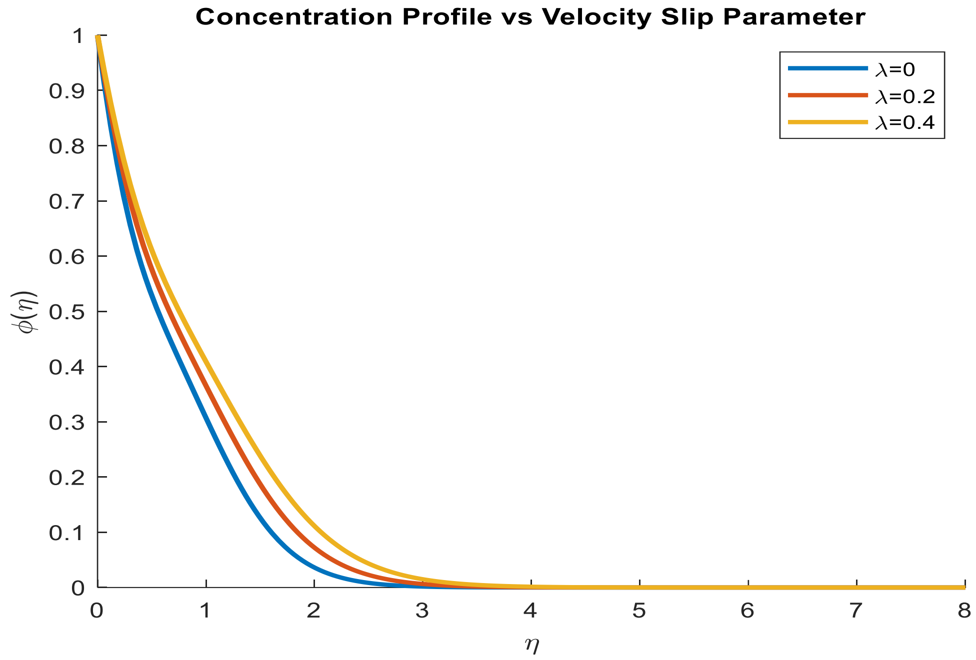

Figure 7: Effect of slip parameter on concentration profile.

The Brownian motion parameter characterizes the random motion of nanoparticles within the base fluid caused by continuous collisions between fluid molecules and suspended particles. It plays an important role in influencing both the temperature and concentration distributions in nanofluid flow. As the Brownian motion parameter increases, the intensified random movement of nanoparticles enhances microscopic mixing within the fluid, which improves thermal energy transport across the boundary layer. Consequently, the temperature profile increases with increasing values of the Brownian motion parameter. This behavior is illustrated in Figure 8, where stronger Brownian motion leads to a rise in thermal energy within the fluid.

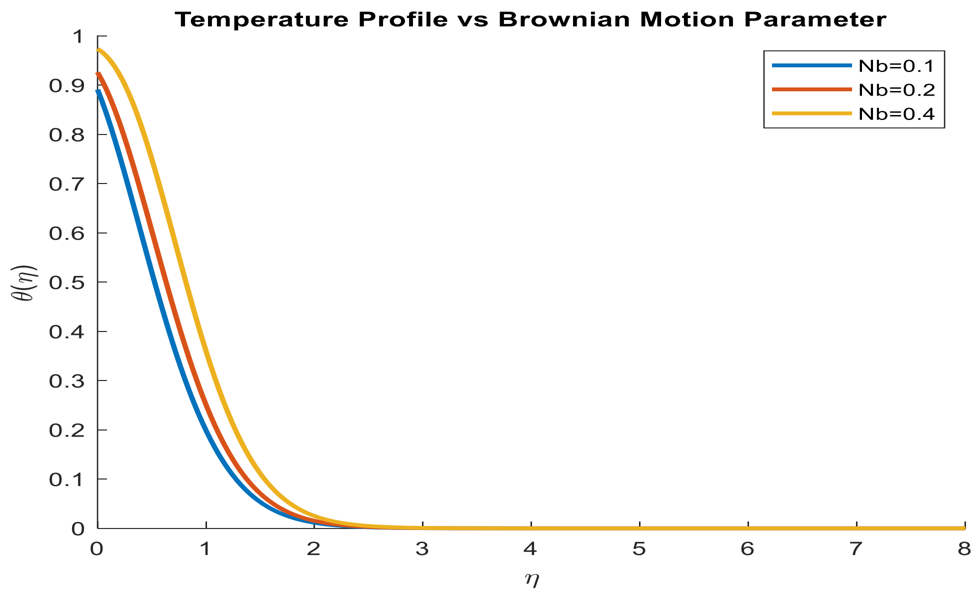

Figure 8: Effect of Brownian motion temperature profile.

An increase in the Brownian motion parameter leads to a reduction in the nanoparticle concentration profile within the boundary layer. This occurs because enhanced Brownian motion intensifies the random movement of nanoparticles, causing them to diffuse more rapidly away from the surface into the surrounding fluid. As a result, the nanoparticle concentration near the stretching sheet decreases. Consequently, the concentration boundary layer thickness becomes thinner with increasing values of the Brownian motion parameter, as illustrated in Figure 9.

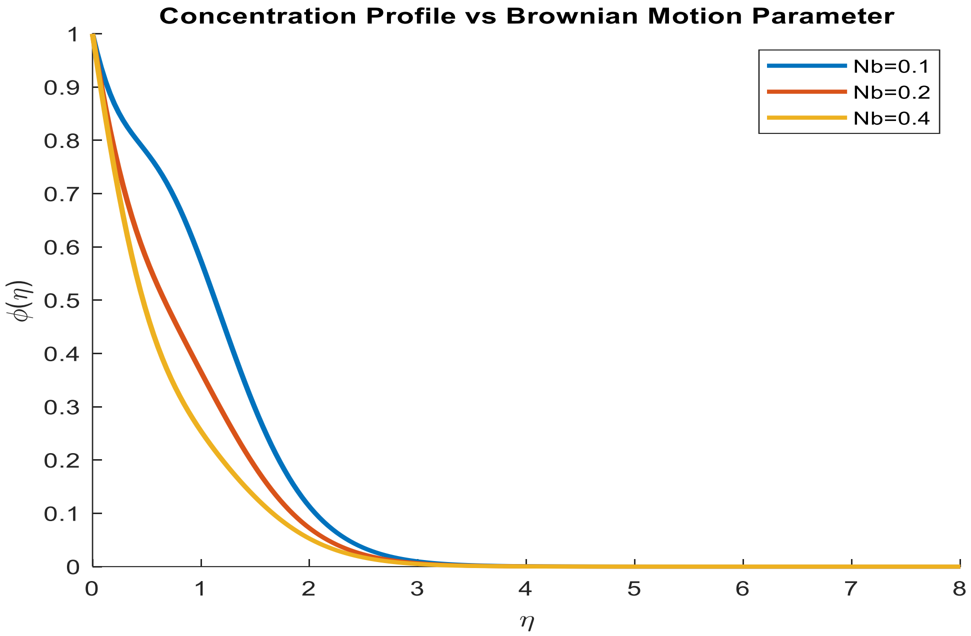

Figure 9: Effect of Brownian motion on concentration profile.

4. Concluding remarks.

The present study investigates the influence of key transport parameters on magnetohydrodynamic nanofluid flow over a stretching sheet with slip boundary conditions by numerically solving the transformed similarity equations using a boundary-value approach.

- The applied magnetic field significantly suppresses the fluid velocity due to the action of the Lorentz force, which enhances momentum resistance within the boundary layer. In contrast, the temperature and concentration fields increase with increasing magnetic parameter
- The Brownian motion parameter enhances the thermal energy distribution within the boundary layer while reducing nanoparticle concentration near the surface due to intensified random particle movement.
- The thermophoresis parameter plays an important role in thickening both thermal and concentration boundary layers by driving nanoparticles from the heated surface toward the ambient fluid region.
- The inclusion of velocity and thermal slip conditions reduces the interaction between the stretching surface and the fluid, leading to a decrease in wall shear stress and heat transfer rate.

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