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Availability Prediction of Unreliable System under Degradation and Imperfect Repair

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Abstract

In this paper, we study the availability indices of a multi-stage degraded system whose failure may occur at each stage and repair is not fully ensured. If the system fails, with some probability it can be repaired to restore its original position otherwise due to imperfect repair, it fails completely. By considering the state transition rates for the degradation process, the transient state probabilities are determined by taking Laplace transforms of the equations governing the model. Some performance measures such as availability, mean life time and mean time to first failure of multistage degraded system etc., are obtained. The sensitivity analysis is also done to examine the effect of parameters on various performance measures.

Keywords: Availability, Degradation, Imperfect repair, Mean life time, Mean time to first failure.

1. Introduction

The machines are the integral components of any machining system and should be maintained to provide the standard productivity under technical as well as economic constraints. The pace of change in the marketplace for manufactured goods is accelerating, as such the competitive producers who wish to survive and excel in a global marketplace must respond to the fast-paced changes in the technology in order to meet the customer's expectations. This in turn, has put pressure on the system engineers to design the machining system that are easy to maintain and can achieve the goal of high reliability and availability. Moreover, the performance of any machining system is very much affected by the machine failure. Apart from this, the degradation in machines with times cannot be ignored as such while predicting the system performance, the degradation factor should be incorporated in the models. The machine failure may be balanced by providing spare part support or by facilitating better repair or both so that the production may not suffer.

In this paper, we study a system with imperfect repairs by assuming that the system is subject to several stages of degradation. Some performance measures such as availability, mean life time (MLT), and mean times to first failure (MTTFF) are established. Section 2 is devoted to the review of literature related to our study in the paper. In section 3, we describe the model by stating suitable assumptions and notations. The state governing equations are provided in section 4. In section 5, the queue size distribution for both transient and steady state models are provided. To validate the analytical results, numerical illustration has been provided in section 6. Finally, conclusions are drawn in section 7.

2. Review of Literature

The reliability of the system decreases from one degradation stage to next degradation stage and might take faster rate as system reaches the last stage of degradation. In this direction, some works have been reported in the literature on the queueing and reliability models of the machine repair problems by incorporating the concept that the machine may undergo degraded failure. Najjar and Gaudiot (1991) have done the scalability analysis of degraded machining systems. Pham et al. (1996) obtained some useful performance measures of K-out of-N complex systems with components subjected to multiple stages of degradation. Park et al. (2000) developed the maintenance policy of a machining system which may subject to slow degradation. Bi-level control of degraded machining system with warm standbys, setup and vacation has been investigated by Jain et al. (2004). Jain and Maheshwari (2005) examined the behavior of (m, M) machining system with spares, reneging and degraded failure. Jain and Mishra (2006) have investigated a multistage degraded machining system with common cause shock failure and state dependent rates. The evaluation of full and degraded mission reliability and mission dependability for intermittently operated, multi-functional systems was considered by Sols et al. (2007). El- Damcese (2009) did the analysis of warm standby systems subject to common-cause failures with time varying failure and repair rates of warm standby. Jain at al. (2010) made availability analysis of embedded computer system with two types of failure and common cause failure. Ke and Wu (2012) proposed multi-server machine repair model with standbys and synchronous multiple vacation. Jain et al. (2012) suggested a multi-state degraded system with inspection and maintainability. Jain (2013) performed transient analysis of machining system with service interruption, mixed standbys and priority. Ke at al. (2013) studied an infinite multi-server queue with an optional service.

In most of the models available for the reliability analysis of machine repair problems, it was assumed that either system can not be repaired after a failure, or maintainability will restore back the system as good as new one. There are some machining systems wherein in case of breakdown, the machines are either replaced or after repair become as good as before failure or fail completely. The feature of imperfect repair has been included by a few researchers in their study on machine repair problems in different contexts. Moustafa (1997) and Amari et al. (1999) have done the reliability analysis of machining system subject to imperfect fault coverage. Amari et al. (2004) designed a K-out of-N:G system subject to imperfect fault coverage. Li et al. (2005) suggested the reliability estimation and performance prediction of multi-state component and coherent system for two parallel machines with an availability constraint. The reliability of fault-tolerant systems with parallel task processing and hierarchical computer based systems subject to common cause failures has been investigated by Xing et al. (2007). The multi-state systems with multi-fault coverage were studied by Levitin and Amari (2008). The fault level coverage (FLC) has been studied for single-phase systems by Myers (2008) and Myers (2009). Levitin and Amari (2010) investigated the time-to-failure distribution of k-out-of-n system with shared standby elements. Peng et al. (2011) discussed the reliability modeling and optimization issues of the multi state systems subject to multi fault coverage. Jain et al. (2012) investigated fuzzy reliability evaluation of a repairable system with imperfect coverage, reboot and common cause shock failure. Levitin et al. (2013) suggested the reliability of redundant systems by incorporating the concept of imperfect fault coverage. Xiang et al. (2013) developed the imperfect fault coverage models by considering the coverage of faulty components but they have not considered the coverage of irrelevant

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(i.e.operational) components. Jain and Rani (2013) did availability analysis of repairable system with warm standby, switching failure and reboot delay.

3. Model Description

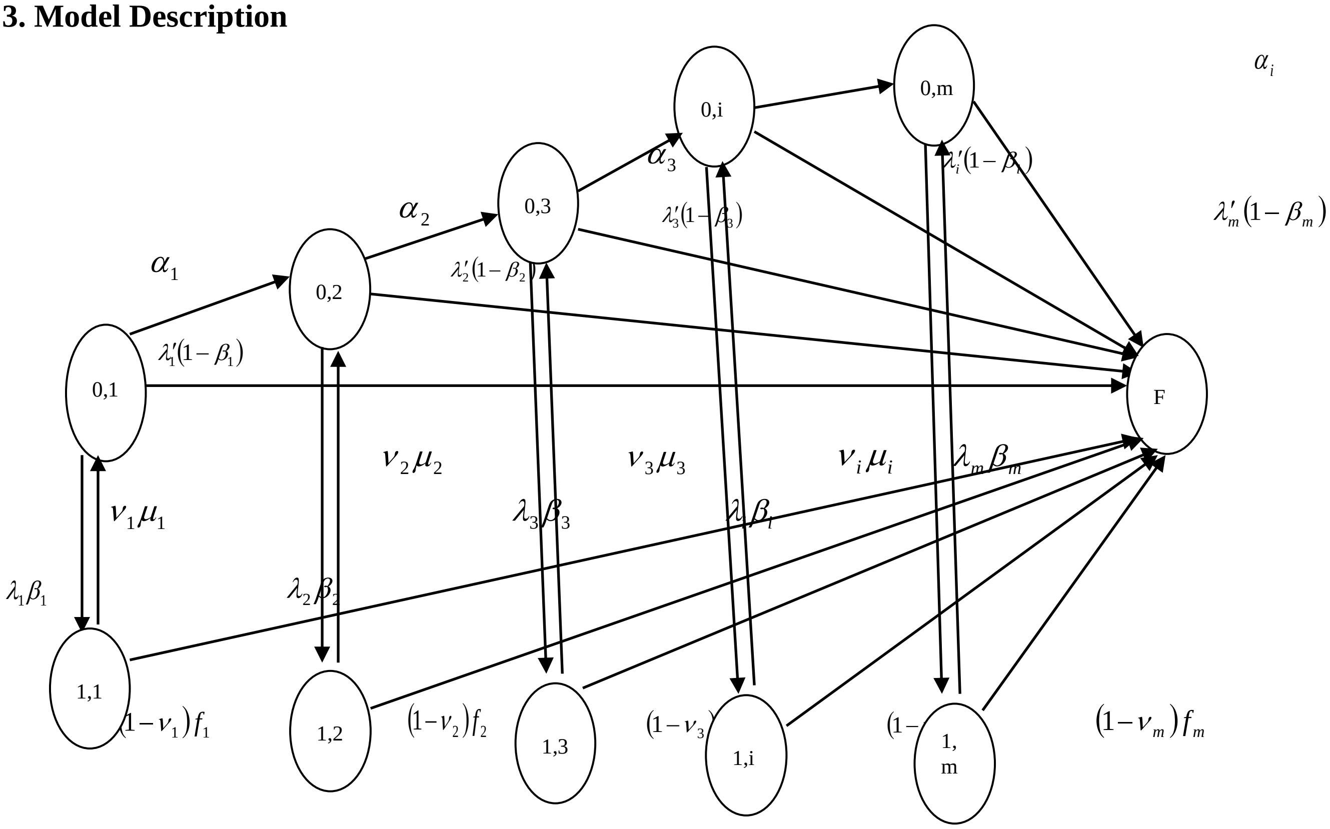

Fig. 1. Transition flow diagram

Consider a machining system having m stages of degradation. It is assumed that the degradation and repair rates depend upon the state of the system. We assume that the machine is in good state (i.e. state '0') initially. After sometime, from its good state the machine reaches to first degraded state upon degradation with rate α_1. From state '0', the machine either can go to a partial failed state with probability β_1 with rate λ_1, or attains a completely failed state with probability (1-β_1). When the machine fails partially, it is either recovered back with probability ν_1 to the good state with rate μ_1 or does not recover with probability $(1-\nu_1)$. If the system is not recovered, it fails completely with rate f_1. This feature is known as imperfect repair.

If the system reaches its first degraded state then upon further degradation, it can either go to the second degraded state with rate α_2 or reaches either with probability $(1-\beta_2)$ to a completely failed state or a partial failed state with probability β_2. Upon partial failure in the second degradation state, either the system is recovered with probability ν_2 or is failed completely with probability $(1-\nu_2)$. This process is continued until the system reaches its m[th] stage of degradation. At the m[th] stage of degradation, the system either fails partially with probability β_m or fails completely with rate λ_m' with probability $(1-\beta_m)$. When the system fails partially at this stage, it is either restored back with probability ν_m or fails completely with rate f_m and with probability $(1-\nu_m)$. At the completely failed state, the system does not recover back and stops operating. The transition flow diagram for different states of the system is shown in fig. 1.

We define the system state probabilities as

$P_{0,i}(t)$: Probability that the system is under i[th] (i=1, 2,…,m) degraded state and is operable.

$P_{1,i}(t)$: Probability that the system is in the i[th] (i=1, 2, …, m) degraded state and is partially failed.

4. Governing Equations and Analysis

Using the birth death process and appropriate rates as depicted in fig.1, we first construct the Chapman- Kolmogorov equations, then after taking Laplace transform of these equations, we shall obtain the Laplace transform of probability of each state of the system. The time dependent probabilities associated with individual states of the system are obtained as follows:

$$\frac{dP_{0,1}(t)}{dt} = -[\alpha_1 + \lambda_1\beta_1 + \lambda_1'(1-\beta_1)]P_{0,1}(t) + \nu_1\mu_1 P_{1,1}(t), \qquad (1)$$

$$\frac{dP_{0,i}(t)}{dt} = -[\alpha_i + \lambda_i\beta_i + \lambda_i'(1-\beta_i)]P_{0,i}(t) + \nu_i\mu_i P_{1,,i}(t) + \alpha_{i-1}P_{0,i-1}(t), \quad \text{i=2,3,…, m-1} \qquad (2)$$

$$\frac{dP_{0,m}(t)}{dt} = -[\lambda_m\beta_m + \lambda_m'(1-\beta_m)]P_{0,m}(t) + \nu_m\mu_m P_{1,m}(t) + \alpha_{m-1}P_{0,m-1}(t) \qquad (3)$$

$$\frac{dP_{1,i}(t)}{dt} = -[\nu_i\mu_i + f_i(1-\nu_i)]P_{1,i}(t) + \lambda_i\beta_i P_{0,i}(t),\ 1 \le i \le m \qquad (4)$$

$$\frac{dP_F(t)}{dt} = \sum_{i=1}^{m} \lambda_i'(1-\beta_i)P_{0,i}(t) + \sum_{i=1}^{m} [(1-\nu_i)f_i]P_{1,i}(t) \qquad (5)$$

Taking Laplace transforms of equations (1)-(4) and after simplifications, we obtain

$$1 = [s + \alpha_1 + \lambda_1\beta_1 + \lambda_1'(1-\beta_1)]P_{0,1}(s) - \nu_1\mu_1 P_{1,1}(s) \qquad (6)$$

$$0 = [s + \alpha_i + \lambda_i\beta_i + \lambda_i'(1-\beta_i)]P_{0,i}(s) - \nu_i\mu_i P_{1,,i}(s) - \alpha_{i-1}P_{0,i-1}(s)\ \text{,i=2,3,…,m-1} \qquad (7)$$

$$0 = [s + \lambda_m\beta_m + \lambda_m'(1-\beta_m)]P_{0,m}(s) - \nu_m\mu_m P_{1,m}(s) - \alpha_{m-1}P_{0,m-1}(s) \qquad (8)$$

$$0 = [s + \nu_i\mu_i + f_i(1-\nu_i)]P_{1,i}(s) - \lambda_i\beta_i P_{0,i}(s), \qquad 2 \le i \le m \qquad (9)$$

Denote
$[\nu_i\mu_i + f_i(1-\nu_i)] = a_i$ and $[\alpha_i + \lambda_i\beta_i + \lambda_i'(1-\beta_i)] = b_i$.

Let u_i and v_i $(1 \le i \le m)$ be the roots of the characteristic equation

$$s^2 + (a_i + b_i)s + a_i b_i - \lambda_i\beta_i\nu_i\mu_i = 0 \quad (10)$$

Then

$$u_i = \frac{(a_i+b_i) - \sqrt{(a_i+b_i)^2 - 4(a_i b_i - \lambda_i\beta_i\nu_i\mu_i)}}{2}$$

and

$$v_i = \frac{(a_i+b_i) + \sqrt{(a_i+b_i)^2 - 4(a_i b_i - \lambda_i\beta_i\nu_i\mu_i)}}{2}$$

On solving equations (6)-(9), we obtain

$$P_{0,1}(s) = \frac{a_1+u_1}{(s-u_1)(u_1-v_1)} + \frac{a_1+v_1}{(s-v_1)(v_1-u_1)} \quad (11)$$

$$P_{0,2}(s) = \frac{(a_1+u_1)(a_2+u_1)\alpha_1}{(s-u_1)(u_1-v_1)(u_1-u_2)(u_1-v_2)} + \frac{(a_1+v_1)(a_2+v_1)\alpha_1}{(s-v_1)(v_1-u_1)(v_1-u_2)(v_1-v_2)} + \frac{(a_1+u_2)(a_2+u_2)\alpha_1}{(s-u_2)(u_2-u_1)(u_2-v_1)(u_2-v_2)} + \frac{(a_1+v_2)(a_2+v_2)\alpha_1}{(s-v_2)(v_2-u_1)(v_2-v_1)(v_2-u_2)} \quad (12)$$

$$\begin{aligned} P_{0,3}(s) = {} & \frac{(a_1+u_1)(a_2+u_1)(a_3+u_1)\alpha_1\alpha_2}{(s-u_1)(u_1-v_1)(u_1-u_2)(u_1-v_2)(u_1-u_3)(u_1-v_3)} \\ & + \frac{(a_1+v_1)(a_2+v_1)(a_3+v_1)\alpha_1\alpha_2}{(s-v_1)(v_1-u_1)(v_1-u_2)(v_1-v_2)(v_1-u_3)(v_1-v_3)} \\ & + \frac{(a_1+u_2)(a_2+u_2)(a_3+u_2)\alpha_1\alpha_2}{(s-u_2)(u_2-u_1)(u_2-v_1)(u_2-v_2)(u_2-u_3)(u_2-v_3)} \\ & + \frac{(a_1+v_2)(a_2+v_2)(a_3+v_2)\alpha_1\alpha_2}{(s-v_2)(v_2-u_1)(v_2-v_1)(v_2-u_2)(v_2-u_3)(v_2-v_3)} \\ & + \frac{(a_1+u_3)(a_2+u_3)(a_3+u_3)\alpha_1\alpha_2}{(s-u_3)(u_3-u_1)(u_3-v_1)(u_3-u_2)(u_3-v_2)(u_3-v_3)} \\ & + \frac{(a_1+v_2)(a_2+v_3)(a_3+v_3)\alpha_1\alpha_2}{(s-v_3)(v_3-u_1)(v_3-v_1)(v_3-u_2)(v_3-v_2)(v_3-u_3)} \end{aligned} \quad (13)$$

In general,

$$P_{0,i}(s) = \sum_{j=1}^{i} \frac{A_{ij}}{(s-u_j)} + \sum_{j=1}^{i} \frac{B_{ij}}{(s-v_j)}; \quad i=1,2,\ldots,m. \tag{14}$$

where

$$A_{ij} = \frac{\prod_{n=1}^{i-1} \alpha_n \prod_{k=1}^{i}(u_j + a_k)}{\prod_{\substack{k=1\\k\neq j}}^{i} (u_j - u_k) \prod_{h=1}^{i}(u_j - v_h)}; \quad B_{ij} = \frac{\prod_{n=1}^{i-1} \alpha_n \prod_{i=1}^{i}(v_j + a_k)}{\prod_{\substack{k=1\\k\neq j}}^{i} (v_j - v_k) \prod_{h=1}^{i}(v_j - u_h)}$$

for $i = 1,2,\ldots.,m$, $\quad j = 1,2,..,m$

Similarly $$P_{1,1}(s) = \frac{\lambda_1\beta_1}{(s-u_1)(u_1-v_1)} + \frac{\lambda_1\beta_1}{(s-v_1)(v_1-u_1)} \tag{15}$$

$$P_{1,2}(s) = \frac{\lambda_2\beta_2\alpha_1(a_1+u_1)}{(s-u_1)(u_1-v_1)(u_1-u_2)(u_1-v_2)} + \frac{\lambda_2\beta_2\alpha_1(a_1+v_1)}{(s-v_1)(v_1-u_1)(v_1-u_2)(v_1-v_2)} + \frac{\lambda_2\beta_2\alpha_1(a_1+u_2)}{(s-u_2)(u_2-u_1)(u_2-v_1)(u_2-v_2)} + \frac{\lambda_2\beta_2\alpha_1(a_1+v_2)}{(s-v_2)(v_2-u_1)(v_2-v_1)(v_2-u_2)} \tag{16}$$

$$\begin{aligned} P_{1,3}(s) = {} & \frac{(a_1+u_1)(a_2+u_1)\lambda_3\beta_3\alpha_1\alpha_2}{(s-u_1)(u_1-v_1)(u_1-u_2)(u_1-v_2)(u_1-u_3)(u_1-v_3)} + \\ & \frac{(a_1+v_1)(a_2+v_1)\lambda_3\beta_3\alpha_1\alpha_2}{(s-v_1)(v_1-u_1)(v_1-u_2)(v_1-v_2)(v_1-u_3)(v_1-v_3)} + \\ & \frac{(a_1+u_2)(a_2+u_2)\lambda_3\beta_3\alpha_1\alpha_2}{(s-u_2)(u_2-u_1)(u_2-v_1)(u_2-v_2)(u_2-u_3)(u_2-v_3)} + \\ & \frac{(a_1+v_2)(a_2+v_2)\lambda_3\beta_3\alpha_1\alpha_2}{(s-v_2)(v_2-u_1)(v_2-v_1)(v_2-u_2)(v_2-u_3)(v_2-v_3)} + \\ & \frac{(a_1+u_3)(a_2+u_3)\lambda_3\beta_3\alpha_1\alpha_2}{(s-u_3)(u_3-u_1)(u_3-v_1)(u_3-u_2)(u_3-v_2)(u_3-v_3)} + \\ & \frac{(a_1+v_3)(a_2+v_3)\lambda_3\beta_3\alpha_1\alpha_2}{(s-v_3)(v_3-u_1)(v_3-v_1)(v_3-u_2)(v_3-v_2)(v_3-u_3)} \end{aligned} \tag{17}$$

In general,

$$P_{1,i}(s) = \sum_{j=1}^{i} \frac{C_{ij}}{(s-u_j)} + \sum_{j=1}^{i} \frac{D_{ij}}{(s-v_j)}; \quad i=1,2,\ldots,m. \tag{18}$$

where

$$C_{ij}=\frac{(\lambda_i\beta_i)\prod_{k=1}^{i-1}\left[(u_j+a_k)\right]\prod_{n=1}^{i-1}\alpha_n}{\prod_{\substack{k=1\\k\neq j}}^{i}(u_j-u_k)\prod_{h=1}^{i}(u_j-v_h)},\quad D_{ij}=\frac{(\lambda_i\beta_i)\prod_{k=1}^{i-1}\left[(v_j+a_k)\right]\prod_{n=1}^{i-1}\alpha_n}{\prod_{\substack{k=1\\k\neq j}}^{i}(v_j-v_k)\prod_{h=1}^{i}(v_j-u_h)},$$

for $i=1,2,....,m$, $j=1,2,..,m$.

Taking inverse Laplace transforms of equations (10)-(18), we obtain

$$P_{0,i}(t)=\left[\sum_{j=1}^{i}A_{ij}e^{-u_j(t)}+\sum_{n=1}^{i}B_{ij}e^{-v_j(t)}\right];\ i=1,2,...,m \tag{19}$$

$$P_{1,i}(t)=\left[\sum_{l=1}^{i}C_{ij}e^{-u_j(t)}+\sum_{n=1}^{i}D_{ij}e^{-v_j(t)}\right]\text{i=1,2,…,m.} \tag{20}$$

5. Performance Measures

The performance evaluation by formulating the indices in terms of probabilities evaluated is the main objective to develop any queueing model of real time system. In order to know the grade of service and efficiency, for the model developed in previous section, now we establish various performance measures as follows:

- System availability is obtained using

$$A(t)=\sum_{i=1}^{m}P_{0,i}(t)=\sum_{i=1}^{m}\sum_{j=1}^{i}\left(A_{ij}e^{-u_j(t)}+B_{ij}e^{-v_j(t)}\right) \tag{21}$$

The system unavailability is given by

$$U(t)=1-A(t) \tag{22}$$

- System unavailability due to individual failures is obtained by using

$$D(t)=\sum_{i=1}^{m}P_{1,i}(t)=\sum_{i=1}^{m}\sum_{j=1}^{i}\left(C_{ij}e^{-u_j(t)}+D_{ij}e^{-v_j(t)}\right) \tag{23}$$

- The time to complete failure of the system i.e the total operational time can be obtained using

$$F(t)=1-A(t)-D(t)$$

$$=1-\sum_{i=1}^{m}\left[\sum_{j=1}^{i}\left(A_{ij}+C_{ij}\right)e^{-u_j(t)}+\sum_{j=1}^{i}\left(B_{ij}+D_{ij}\right)e^{-v_j(t)}\right] \tag{24}$$

- The expected operational time is given by

$$EOT(t) = \int_0^t \sum_{i=1}^{m} P_{0,i}(x)dx = \int_0^t \sum_{i=1}^{m}\left[\sum_{j=1}^{i} A_{ij}e^{-u_j(x)} + \sum_{j=1}^{i} B_{ij}e^{-v_j(x)}\right]dx$$

$$= \sum_{i=1}^{m}\left[\sum_{j=1}^{i}\frac{1}{u_j}A_{ij}(1-e^{-u_j(t)}) + \frac{1}{v_j}\sum_{j=1}^{i} B_{ij}(1-e^{-v_j(t)})\right] \quad (25)$$

- The mean life time (MLT) of the system is

$$MLT = \int_0^{\infty}[1-F(t)]dt = \int_0^{\infty}[A(t)+D(t)]dt \quad (26)$$

- The mean operation life time (MOLT) of the system is given by

$$\text{MOLT} = \int_0^{\infty} A(t)dt = \sum_{i=1}^{m}\sum_{j=1}^{i}\left(\frac{A_{ij}}{u_j} + \frac{B_{ij}}{v_j}\right) \quad (27)$$

- The mean time to first failure is given by

$$MTTFF = \int_0^{\infty} R(t)dt \quad (28)$$

6. Numerical Illustration

In this section, the sensitivity analysis is carried out to demonstrate the effect of parameters λ_1, λ_2, μ_1, μ_2, β_1, β_2, α_1, α_2, f and α on various performance measures such as availability (A(t)), mean life time (MLT) and mean operation life time (MOLT). The results are calculated using MATLAB software and are summarized in tables 1-3 and figs 1-3. For computation purpose, we consider two sets of degradation rates (α_1, α_2). We set default parameters as

λ_1=0.01, λ_2=0.02, λ_1' = 0.001, λ_2' = 0.002, β_1=1, β_2= 2, μ_1=0.5, μ_2= 0.9, ν_1= 0.1, ν_2= 0.06, f_1= 0.009, f_2= 0.008 for tables 1-3 and $\lambda_1 = 0.01\lambda$, $\lambda_2=0.02\lambda$, $\lambda_1' = 0.001\lambda$, $\lambda_2' = 0.002\lambda$, $\beta_1 =1$, $\beta_2 = 2$, $\mu_1 = 0.5\mu$, $\mu_2 = 0.9\mu$, $\nu_1 = 0.1\nu$, $\nu_2 = 0.06\nu$, $f_1 = 0.009f$, $f_2 = 0.008f$, $\alpha_1 = 0.001\alpha$, $\alpha_2 = 0.002\alpha$, $\lambda = 0.1$, $\beta = 0.1$, $\mu = 1$, f = 0.1, $\nu = 0.1$, $\alpha = 0.5$, for figs 1-3.

For two sets of (α_1, α_2) in table 1, we display the variation in MLT and MOLT for different values of μ_1 and μ_2. It is clear from this table that both MLT and MOLT increase on increasing μ_1 and μ_2. The effect of failure rates λ_1 and λ_2 can be visualized in table 2. Table 2 depicts that both MLT and MOLT decrease with the increase in λ_1 and λ_2. This feature can be observed in many machining systems. Table 3 shows the effect of β_1 and β_2 on MLT and MOLT. It is noted from this table that both MLT and MOLT decrease on increasing β_1 and β_2. Moreover, from tables 1-3, we find that both MLT and MOLT increase with the increase in degraded failure rates α_1 and α_2 which is same what we expect in machining systems.

In fig. 1, we illustrate the effect of failure rate f of the operating units on the availability by varying time t. We observe from this figure that the availability decreases by a constant value when f increases for different values of t. Fig. 2 depicts the effect of degradation rate (α) on the availability for increasing values of time t. We note that the availability first decreases sharply on increasing α for lower values of t and then after becomes almost constant for higher values of t. The effect of failure rate λ of the operating units on the availability for increasing values of time t can be observed from fig. 3. It can be seen from fig. 3 that the availability first decreases slowly for lower values of t and then after decreases sharply for higher values of t.

Overall, based on numerical results evaluated by taking an illustration for our model, we have the following observations-

- The availability of the machining system can be kept high by incorporating low failure rate/degraded failure rate and high repair rate.
- The life time of the machines can be increased by providing the better repair facility so that the repair of the failed machines can be done with high rate.
- As we expect, availability is a decreasing function of time. Thus while evaluating the reliability/availability of the machining system, the ageing factor should be taken into consideration as it has a significant effect on the performance of the system.

7. Conclusion

In industrial organizations operated in machining environment, multiple degradation of the system may lead to overall availability down to an unexpected level. In the present investigation, we have investigated a machining system which undergoes multiple stages of degradation. There is a provision of repair in case of partial failure of the system. To make our model to depict more realistic scenario of machining system, the concept of imperfect repair is also incorporated. Some useful performance measures in explicit form may be used for the performance prediction and will be helpful to achieve high reliability/availability of the real time systems. It is suggested that the system designers must follow a maintainability schedule in order to avoid the partial failure. The model developed provides an insight how to design more reliable and effective embedded systems such as computer, communication and flexible manufacturing systems etc., wherein degradation and failures are unavoidable factors.

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