Journal of Advances in Developmental Research (IJAIDR)
E-ISSN: 0976-4844 ● Website: www.ijaidr.com ● Email: editor@ijaidr.com

Status of Magnetic Monopoles and Their Role in Quark Confinement

Chandrabhan Singh

NET- Physics
chandrabhansingh2032@gmail.com

Abstract
Quark confinement is the strangest and most important mystery of high energy physics. Different models have been proposed to solve it among which magnetic monopoles are promising candidates. In this paper we will discuss some theoretical and experimental studies, which advocates the existence of magnetic monopoles and provide us a vision to understand quark confinement.

Keywords: Quark Confinement, Monopoles, Abelian Projection, Magnetic Symmetry, Lattice QCD

1. Introduction

The dual superconductor picture is one of the most important ideas in the theory of color confinement [1] which sets a striking parallelism between a normal superconductor and QCD vacuum, explains that the color flux tube between quarks is caused by the condensation of color magnetic monopoles in the QCD vacuum through the dual Meissner effect [2, 3]. Although the presence of monopoles and their role as the endpoints of flux tubes provides a mechanism for the confinement of quarks within hadrons but till yet QCD does not appear as a reliable candidate of color magnetic monopoles. Theoretical developments in the understanding of monopoles motivates for the existence of monopole. Dirac in 1931 [4] observed that the existence of monopoles would necessitate the quantization of electric charge. 't Hooft and Polyakov [5, 6] discovered that magnetic monopoles will occur naturally in a Georgi-Glashow model, where the non-Abelian local symmetry is broken down by the Higgs mechanism into an Abelian symmetry.

To study the quark confinement using dual superconductor mechanism two methods mostly used are Abelian projection method [2, 3] and Field decomposition method [7] discussed in next sections. These methods have been supported by numerical simulations, and have provided qualitative insights into the nature of quark confinement. The aim of this comprehensive review is to provide a thorough summary of the current state of knowledge regarding the elusive concept of magnetic monopoles.

2. Theory of magnetic monopoles

The section would likely start by introducing the theoretical frameworks in which magnetic monopoles are either permitted or strongly suggested to exist.

2.1 Monopoles in classical electromagnetic theory

In classical electromagnetic theory, monopoles are not present, and the concept of a magnetic monopole is not consistent with the equations of electromagnetism as formulated by Maxwell [8] can be written in the form.

$$\nabla.\vec{E} = \rho_E, \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}, \nabla \times \vec{B} = \vec{J_E} + \frac{\partial \vec{E}}{\partial t}$$

Where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields respectively, ρ_E is the electric charge density and $\vec{J_E}$ is the electric current density. If no electric charges are present then ρ_E and $\vec{J_E}$ vanish and the equation (1) exhibit the symmetry known as electromagnetic duality which involves interchanging the electric field $\vec{E}$ and the magnetic field $\vec{B}$ while changing the sign of one of them as $\vec{E} \to \vec{B}$ and $\vec{B} \to -\vec{E}$. Thus the existence of electric charge breaks the symmetry. It had seen that symmetry can be restore by assumption of the existence of magnetic charge along with the duality condition. A new set of equations with a profound symmetry is given as.

$$\nabla.\vec{E} = \rho_E, \qquad \nabla \cdot \vec{B} = \rho_M \tag{1}$$

$$-\nabla \times \vec{E} = \vec{J_M} + \frac{\partial \vec{B}}{\partial t}, \qquad \nabla \times \vec{B} = \vec{J_E} + \frac{\partial \vec{E}}{\partial t} \tag{2}$$

Where ρ_M and $\vec{J_M}$ are the magnetic charge density and the magnetic current density respectively characterised by the replacement of $\vec{J_E} \to \vec{J_M}$, $\vec{J_M} \to -\vec{J_E}$ and $\rho_E \to \rho_M$, $\rho_M \to -\rho_E$. The fundamental equations governing electromagnetic fields maintain their structure when represented in the complex domain. Combination of the electric and magnetic fields into a single complex field, $\vec{E} + \vec{\imath B}$ remain invariant under complex rotation represented as.

$$\vec{E} + i\vec{B} \to e^{i\theta}(\vec{E} + i\vec{B})$$

$$\rho_E + i\rho_M \to e^{i\theta}(\rho_E + i\rho_M)$$

$$\vec{J_E} + i\vec{J_M} \to e^{i\theta}(\vec{J_E} + i\vec{J_M}) \tag{3}$$

This invariance allows for a convenient and elegant way to analyze and solve problems involving time-varying electromagnetic fields. However magnetic monopoles are not a part of classical electromagnetism, but one can explore the idea that the duality transformation under Maxwell's equations suggests an elegant way in which magnetic monopoles could fit into the framework of classical electromagnetism. In classical electromagnetism, the electric field and magnetic field can be expressed in terms of the scalar potential (φ) and vector potential ($\vec{A}$) as.

(4)
$$\vec{E} = -\nabla\Phi - \frac{\partial\vec{A}}{\partial t}, \qquad \vec{B} = \nabla\times\vec{A}$$

$$\rho_M = \nabla\cdot\left(\nabla\times\vec{A}\right) = 0$$

However, equation $\vec{B} = \nabla\times\vec{A}$ does not favour the existence of magnetic monopole as $\rho_M = \nabla\cdot\vec{B}$ and hence

But the scalar and vector potential are not physical objects themselves so there is no fundamental theoretical obstacle to including magnetic monopoles in the classical electromagnetic theory. In this extended framework, the theory remains consistent and mathematically elegant, even with the inclusion of magnetic charges.

2.2 Monopoles in quantum theory

In the context of quantum theory, monopoles are postulated to carry quantized magnetic charges, analogous to the quantization of electric charge. In some quantum field theories monopoles are closely associated with topological defects and support the fact that quantum mechanics is consistent with the existence of magnetic monopoles.

2.2.1 The Dirac monopole

Dirac's solution to the problem of existence of monopole was to introduce a singularity in the vector potential. This singularity can be thought of as a "Dirac string." The Dirac string is an infinitesimally thin, infinitely long line that extends from the location of the monopole to infinity shown in figure 1 [9]. Along this string, the vector potential becomes singular, and it can be visualized as a magnetic analog of an infinitely long, infinitesimally thin solenoid with no end.

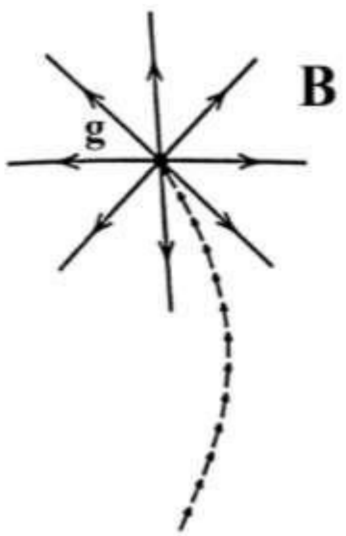

Fig 1: Representation of magnetic field produced by a magnetic monopole g as the end of a Dirac string i.e. line of dipoles or a tightly wound solenoid that stretches off to infinity

Dirac assumed the monopole of strength ‘g’ located at the origin. The static magnetic field produced by point magnetic monopole analogous to a point electric charge is Coulomb type and in radial direction is given by

$$\vec{B} = \frac{g}{r^2}\hat{r} \quad (5)$$

By taking the divergence of magnetic field $\vec{B}$, where we have used $\nabla.\frac{\hat{r}}{r^2} = 4\pi\delta^3(r)$, and magnetic charge is at the origin, it follows.

$$\nabla.\vec{B} = 4\pi g\delta^3(r) \quad (6)$$

The total magnetic flux through the sphere of radius ‘r’ surrounding the origin is given by

$$\Phi = \oint \vec{B}.d\vec{A} = 4\pi r^2 B = 4\pi g \quad (7)$$

For Quantum mechanical description of Dirac monopole consider a time independent electrically charged particle ‘e’ in a magnetic field of monopole. Dirac noted that the interaction of an electric charge with a vector potential $\vec{A}$ is given by the phase in the wave function. The wave function of the free particle is given by.

$$\psi(r) = \psi_0(r)exp\left[\frac{ie}{\hbar c}\int \overrightarrow{dr}.\vec{A}\right] \quad (8)$$

Where $\psi_0(r)$is the solution of the free Schrödinger equation. Applying momentum operator on wavefunction.

$$\vec{p}\psi(r) = -i\hbar\nabla\psi(r)$$

$$\vec{p}\psi(r) = -i\hbar\nabla\psi_0(r)exp\left[\frac{ie}{\hbar c}\int \overrightarrow{dr}.\vec{A}\right] + \frac{e}{c}\vec{A}exp\left[\frac{ie}{\hbar c}\int \overrightarrow{dr}.\vec{A}\right]\psi_0(r)$$

$$\psi(r) = -i\hbar\nabla\psi_0(r)exp\left[\frac{ie}{\hbar c}\int \overrightarrow{dr}.\vec{A}\right] + \frac{e}{c}\vec{A}\psi(r)$$

$$\left(\vec{p} - \frac{e}{c}\vec{A}\right)\psi(r) = -i\hbar\nabla\psi_0(r)exp\left[\frac{ie}{\hbar c}\int \overrightarrow{dr}.\vec{A}\right] \quad (9)$$

So, the important point is there that presence of vector potential only responsible to change the wave function. In the electromagnetic field $\vec{p} \rightarrow \vec{p} - e\frac{\vec{A}}{c}$, as the wave function changes $\psi(r) \rightarrow \psi_0(r) exp\left[\frac{ie}{\hbar c}\int \overrightarrow{dr}.\vec{A}\right]$. It was observed in quantum theory by Aharonov Bohm effect [10] that the phase of a wave function is not observable, but the difference of phases observed actually. For a charged particle moving in an electromagnetic field, the phase acquired by the wave function depends not only on the kinetic energy of the particle but also on the electromagnetic potential. It allows charged particles to be affected by the vector potential even where the magnetic field is zero, resulting in a change in the complex phase of the particle's wavefunction for objects like a Dirac solenoid. The phase acquired by the wave function as the particle moves along a closed path is given by $\alpha = \frac{ie}{\hbar c}\int \overrightarrow{dr}.\vec{A}$. The accumulated phase difference which is observable quantum mechanically between the two different path at fixed r, θ with ϕ ranging from 0 to 2π is given by.

$$\Delta\alpha = \frac{e}{\hbar c}\oint \vec{A}.\overrightarrow{dr}$$

$$\Delta\alpha = \frac{e}{\hbar c}[Flux\ through\ the\ closed\ surface] \quad (10)$$

$$\Delta\alpha = \frac{e}{\hbar c}\phi_g(r,\theta) \quad (11)$$

Where $\phi_g(r,\theta)$represent flux through the area covered by the cap in the sphere shown by the shaded region in figure 2 [11]. As θ→0, the loop shrinks to a point and flux passing through the shaded region approaches to zero. i.e. $\phi_g(r,0) = 0$.

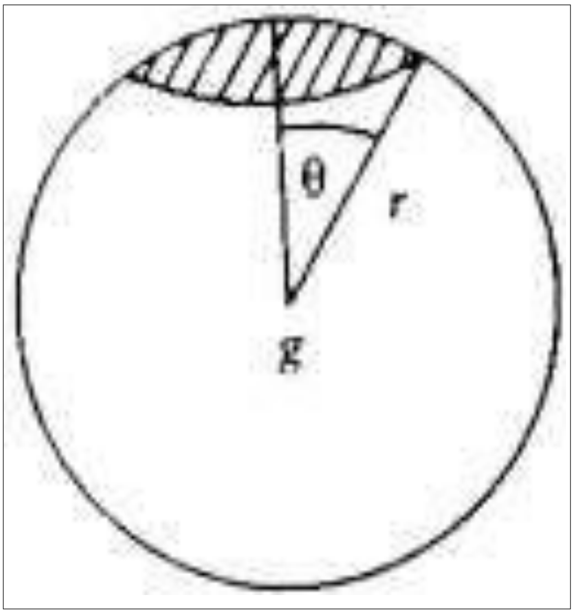

Fig 2: Representation of the flux through the area covered by the cap $\phi_g(r,\theta)$

From equation (10) as θ increases the shaded region increases hence more flux. As θ→π cap should encloses the flux 4 mg.

$$\phi_g(r, \pi) = 4\pi g \quad (12)$$

But as θ → π, the cap again shrinks to a point and $\vec{A}$ become singular along the entire negative z axis. The string must be continuous and can be chosen to be along any direction. This is known as Dirac string. In general Dirac string only observable at.

$$\Delta\alpha = 2n\pi \quad (13)$$

From equation (12) and (13) we have.

$$2n\pi = \frac{e}{\hbar c} 4\pi g \quad (14)$$

Hence, $eg = \frac{n\hbar c}{2}$ (15)

It implies that all charges in nature will be quantised by $\hbar c/2$. It is called the Dirac quantisation condition [4]. Dirac's condition introduces a unique quantization pattern, reflecting the interplay between electric and magnetic charges.

Further Wu and Yang [12, 13] constructed Dirac monopole by offering a different approach. He considered that Dirac string singularity is unphysical and also equation (6) refers that physical singularity should only occur at the origin. Wu and Yang's approach divides the space surrounding the monopole into two different overlapping regions as shown in figure 3, [14] and suggested that rather than introducing a singular vector potential there should be different potentials in different overlapping regions.

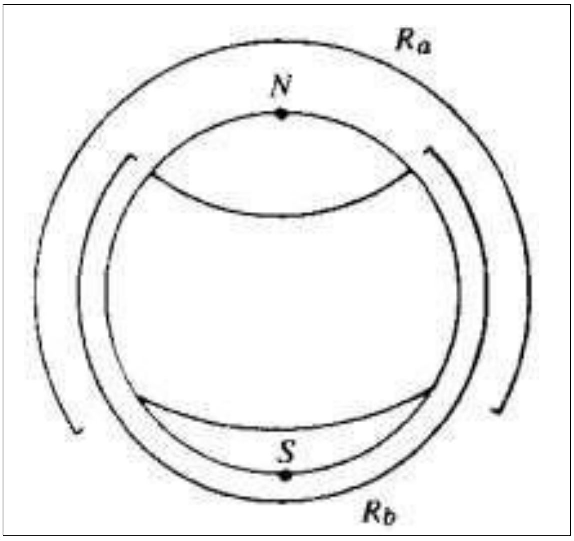

Fig 3: The Wu-Yang configuration describing a magnetic monopole using overlapping region Ra singular along the S pole and Rb singular along the N pole

For overlapping region R_a, Vector potential A to be singular along r = -z, we should have (S -pole).

$$A_r = 0, A_\theta = 0, A_\phi^s = \frac{g(1-\cos\theta)}{r\sin\theta} \quad (16)$$

For overlapping region R_b, Vector potential A to be singular along r = z, we should have (N-pole).

$$A_r = 0, A_\theta = 0, A_\phi^N = \frac{-g(1+\cos\theta)}{r\sin\theta} \quad (17)$$

$A_\Phi^N - A_\Phi^S$ should be zero field and gauge equivalent of some λ, given as.

$$A_\Phi^N - A_\Phi^S = \nabla\lambda = \frac{-2g}{r\sin\theta} \quad (18)$$

If we delete the z axis, then A^N and A^S will give the same magnetic field. A common boundary is taken to be at the equator $\theta = \frac{\pi}{2}$ to calculate total magnetic flux.

$$\Phi = \oint_{\theta=\frac{\pi}{2}} A_\Phi^S . dl_\phi^S - \oint_{\theta=\frac{\pi}{2}} A_\Phi^N . dl_\phi^N \quad (19)$$

Using equation (16) and (17) we get.

$$\Phi = 4\pi g \quad (20)$$

The total magnetic flux for expression of vector potential given in equation (20) represents a valid formulation which favours Dirac monopole.

2.2.2 ‘t Hooft Polyakov monopole

The 't Hooft-Polyakov monopole is a theoretical concept in particle physics, addressing the existence of magnetic monopoles in non-Abelian gauge theories, independently proposed by Gerard 't Hooft and Alexander Polyakov. In this theoretical model, ’t Hooft and Polyakov [5, 6] focused on the SU(2) gauge group, which is a non-Abelian group. The local symmetry of the theory is spontaneously broken down to U (1) by a Higgs field. This breaking of symmetry from SU (2) to U(1) is fundamental to the generation of 't Hooft-Polyakov monopoles. Importantly, 't Hooft-Polyakov monopoles exhibit smooth solutions, in contrast to Dirac monopoles with singular vector potentials. Consider the non Abelian symmetry group O (3). The lagrangian density,

contains the gauge field $F_{\mu\nu}^a$ and an isovector Higgs field ϕ^a is invariant under isospin rotations described by

$$L = \frac{-1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \frac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a) - \frac{m^2}{2}\phi^a\phi^a - \lambda(\phi^a\phi^a)^2 \quad (21)$$

where the gauge field defined as $F^a_{\mu\upsilon} = \partial_\mu A^a_\upsilon - \partial_\upsilon A^a_\mu + e\epsilon^{abc} A^b_\mu A^c_\upsilon$, and the covariant derivative defined as.

$$D_\mu \phi^a = \partial\mu\phi^a + e\varepsilon^{abc} A^b_\mu \phi^c \quad (22)$$

The potential has a minimum for $(m^2 < 0)$ at $|\Phi_0| = \left(\frac{-m^2}{4\lambda}\right)^{\frac{1}{2}} = H$. 't Hooft look for a solution consistent with the asymptotical form. There exist regular solutions to the field equations derived from the Lagrangian given in equation (21) in which the gauge potentials have nontrivial asymptotic form. A general spherically symmetric ansatz for the scalar field is given as

$$\phi^a = H\frac{r^a}{r}, (r \to \infty) \quad (23)$$

Since the solution is a magnetic monopole, we expect the electric field to be zero, so we work in the coulomb gauge $A^a_0 = 0$. At infinity, the vacuum solution depends on the direction.

$$A^a_i = -\epsilon_{iab}\frac{r^b}{er^2}, (r \to \infty) \quad (24)$$

Hence the energy configuration is partitioned into two components, originating from the field inside and outside the core, respectively. Beyond the core, $D_i\phi^a = 0$. Hence at $(r \to \infty)$, ϕ takes on its vacuum value and has to be simple function of r. This leads to a relation between the full non Abelian gauge field strength $F^a_{\mu\upsilon}$ and Abelian part of the full field strength $F_{\mu\upsilon}$. A close look at the U (1) gauge potential, the electromagnetic field $F_{\mu\nu}$ given as.

$$F_{\mu\nu} = \frac{1}{|\Phi|}\phi^a F^a_{\mu\nu} - \frac{1}{e|\Phi|^3}\epsilon_{abc}\phi^a\left(D_\mu\phi^b\right)\left(D_\upsilon\phi^c\right) \quad (25)$$

This $F_{\mu\nu}$ is more complicated than usual definition of Abelian field strength. To reduce it to the usual one boundary conditions applied are.

$$A^3_\mu \equiv A_\mu \neq 0, \phi^{1,2} = 0, \phi^3 = H \neq 0 \quad (26)$$

The Maxwell vector potential now given as.

$$A_\mu = \frac{1}{|\phi|}\phi^a A^a_\mu \quad (27)$$

Hence electromagnetic field calculated as.

$$F_{\mu\nu} = \partial_\mu A_\upsilon - \partial_\upsilon A_\mu - \frac{1}{e|\Phi|^3}\epsilon_{abc}\phi^a\left(D_\mu\phi^b\right)\left(D_\upsilon\phi^c\right) \quad (28)$$

By incorporating the asymptotic conditions expressed in equations (23) and (24), it becomes evident that the Maxwell potential A_μ undergoes cancellation, leading to the emergence of the electromagnetic field solely attributed to the presence of the Higgs field. The asymptotic form of ‘t Hooft Polyakov solution find as.

$$F_{0i} = 0, F_{ij} = \frac{-1}{e r^3} \epsilon_{ijk} r^k \qquad (29)$$

The form (29) shows that the asymptotic electric field vanishes and asymptotic magnetic field that of a monopole is given as

$$B_k = \frac{r^k}{e r^3} \qquad (30)$$

Corresponding to this equation (30) magnetic flux is given by equation (12) is

$$\Phi = \frac{4\pi}{e} \qquad (31)$$

Comparing equation (12) and equation (31) we get $eg = 1$. (32)

This statement indicates that the magnetic charge associated with't Hooft-Polyakov monopole is twice the minimum magnetic charge allowed by the electric charge and the Dirac quantization condition. Hence, the magnetic charge ‘g’ of the monopole achieves its minimum value in accordance with the Dirac condition. This classical solution with finite energy, possessing topological nontriviality, is commonly referred to as the 't Hooft-Polyakov monopole showcases a finite core, eliminating the need for a Dirac string.

3. Monopoles in Quantum Chromodynamics (QCD)

In trying to understand magnetic monopoles in the context of Yang-Mills theory without matter fields, we have come across two significant methods in our further studies. One is the Abelian projection, which is a way of fixing part of the gauge in a certain manner. This process reveals singularities in the resulting gauge fields, which represent important features of the theory. Another method is the gauge field decomposition, which achieves a similar result but without explicitly fixing the gauge.

3.1 Appearance of monopoles in Abelian projected QCD

In this section we delve into the intriguing phenomenon of formation of monopoles and exploring the condensation of monopole particularly in the framework of Abelian gauge fixing [15]. In this context the gluon field undergoes a transformation, developing a singularity in the spatial vicinity of points where Abelian gauge fixing proves unsuccessful. Within these regions magnetic monopole manifest and showcasing that the topological defects of Abelian gauge fixing are sources of magnetic monopoles in the

realm of QCD. In reducing QCD to an Abelian theory by aligning specific components of the gluon field A_μ through gauge transformations, the use of a scalar field is a common technique in quantum field theories to simplify calculations by fixing a particular gauge. To fix a gauge scale field $\phi(x)$ can be written in the form.

$$\phi(x) = \phi_a(x)\tau_a \quad (33)$$

Where $\phi_a(x)$ is scalar field corresponding to the color index 'a' and τ_a are the $N_c^2 - 1$ generators of the $SU(N_c)$ group.

To define different gauges in QCD, local rotations in color space generated by the $SU(N_c)$ group. This orientation of the vector $\phi(x)$ in color space, is referred as a gauge transformation ensures that the physical observables remain unchanged at the space time are generated by the operators of the form

$$\omega(x) = \exp[i\chi_a(x)\tau_a] \quad (34)$$

The operators $\omega(x)$ are the element of the color $SU(N_c)$ group. The generators τ_a are hermitian $N_c \times N_c$ matrices so that the field $\phi(x)$ may be viewed as a traceless matrix in color space. It is possible to perform a gauge transformation to diagonalize the color matrices. Diagonalizing the matrices simplifies the representation of the color fields, making calculations more manageable. In the simplest SU(2) group the gauge transformation $\omega(x)$ brings the field $\phi(x)$ into diagonal form given by.

$$\phi = \phi_a\tau_a \rightarrow \omega\phi\omega^{\dagger} = \lambda\tau_3 = \begin{bmatrix} \lambda & 0 \\ 0 & -\lambda \end{bmatrix} \quad (35)$$

Where $\lambda(x)$ are the eigenvalues of matrix $\phi(x)$ given by $\lambda = \pm\sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2}$.

The monopole occurs at the degeneracy point of the diagonalised elements of $\phi(x)$. The degeneracy point in the Abelian gauge appears as the singular point of $\phi(x)$ like the centre of hedgehog configuration as shown in figure 4 [15].

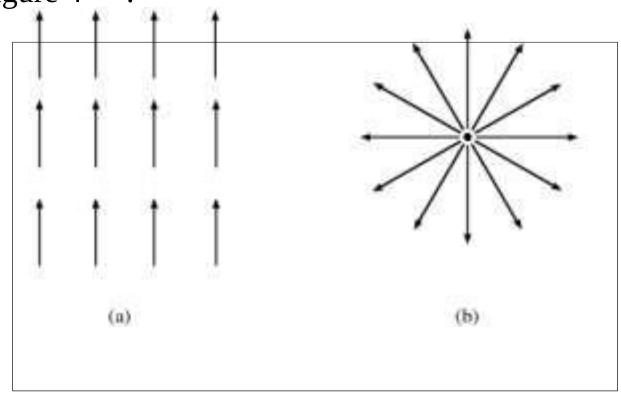

Fig 4: Representation of separation of gluon field variable in SU (2) Abelian gauge. (a) For regular part no monopole appear. (b) Monopole appears for the singular part

Hence the singular point is to satisfy the conditions that all the three components $\phi_{a=1,2,3}(\vec{r})$ should vanish at specific points in the space such that. $\vec{r} = \vec{r_i}$

$$\phi_1(\vec{r_0}) = 0,\ \phi_2(\vec{r_0}) = 0,\ \phi_3(\vec{r_0}) = 0 \qquad (36)$$

All three equations determine the three components (x_0, y_0, z_0) of the vector $\vec{r_0}$. Since at the point $\vec{r_0}$, it is not possible to define the gauge and gluon field develops a singularity at that point.

The topological nature at the vicinity of the point $\vec{r_0}$ explained in terms of Taylor expansion. It gives $\phi(\vec{r})=\phi^a(\vec{r})\tau^a=\tau^a C^{ab}(x - x_0^b)$ (37)

Where C^{ab}defines a coordinate system in rotation with $\phi^a(\vec{r})$given as

$$C^{ab} = \partial^{ab}\phi^a(r_0) \qquad (38)$$

The hedgehog configuration around the singular point of $\phi(\vec{r})$corresponding to the simplest nontrivial topology of the non-trivial homotopy group $\pi_2 SU(2)/U(1)_3 = Z_\infty$, and the Abelian gauge field has the singularity as the monopole appearing from the hedgehog configuration.

Let (r, θ, ϕ) be the spherical coordinates. The hedgehog field configuration is expressed by.

$$\phi = \tau^a r_a = r sin\theta cos\phi\tau^1 + r sin\theta sin\tau^2 + r cos\theta\tau^3$$

$$= \frac{r}{2}\begin{pmatrix} cos\theta & e^{-i\phi} sin\theta \\ e^{i\phi} sin\theta & -cos\theta \end{pmatrix} \qquad (39)$$

Φ can be diagonalized by the gauge transformation with ω as.

$$\omega(\theta, \Phi) = \begin{pmatrix} e^{i\phi} cos\frac{\theta}{2} & sin\frac{\theta}{2} \\ -sin\frac{\theta}{2} & e^{-i\phi} cos\frac{\theta}{2} \end{pmatrix} \qquad (40)$$

Where θ, ϕ denotes the polar and azimuthal angle respectively. Here on the z axis ($\theta = 0$ or $\theta = \pi$), At the positive z axis,$\theta = 0$, and applying it in equation (40) the dependency of ω on ϕ is given as.

$$\omega = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} \qquad (41)$$

By using the gauge transformation scalar field ϕ becomes.

$$\Phi^{\omega} = \omega\Phi\omega^{\dagger} = \frac{r}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = r\tau_3 \qquad (42)$$

Now, the gluon field transformation under the same gauge transformation as defined in equation (34).

$$A_\mu \to A_\mu^{\omega} = \omega\left(A_\mu + \frac{1}{ie}\partial_\mu\right)\omega^{\dagger} \qquad (43)$$

In spherical coordinates the expression of the vector $\omega\partial_\mu\omega^\dagger$ is.

$$\omega\partial_\mu\omega^\dagger = \hat{r}\left(\omega\frac{\partial}{\partial r}\omega^\dagger\right) + \hat{\theta}\left(\omega\frac{\partial}{\partial \theta}\omega^\dagger\right) + \hat{\phi}\frac{1}{r sin\theta}\left(\omega\frac{\partial}{\partial \phi}\omega^\dagger\right) \quad (44)$$

From the equation (40) and (44) we can find.

$$\left(\omega\frac{\partial}{\partial r}\omega^\dagger\right) = 0$$

$$\left(\omega\frac{\partial}{\partial \theta}\omega^\dagger\right) = \frac{1}{2}\begin{pmatrix} 0 & e^{-i\phi} \\ e^{-i\phi} & 0 \end{pmatrix} = -e^{-i\phi}\tau_2$$

$$\left(\omega\frac{\partial}{\partial \phi}\omega^\dagger\right) = \frac{i}{2}\begin{pmatrix} -cos\theta - 1 & e^{i\phi} sin\theta \\ e^{-i\phi} sin\theta & cos\theta + 1 \end{pmatrix}$$

$$= -i(1 + cos\theta)\tau_3^a + isin\theta cos\phi\tau_1^a - isin\theta sin\phi\tau_2^a \quad (45)$$

And hence,

$$\frac{1}{ie}\omega\partial_\mu\omega^\dagger = \frac{1}{e}\left[-\tau_2^g e^{i\phi}\right]\hat{\theta} + \frac{1}{e}\left[\frac{1}{r}\left(cos\phi\tau_1^g - sin\phi\tau_2^g\right)\right]\hat{\phi} + \frac{1}{e}\left[\frac{-(1+cos\theta)}{rsin\theta}\tau_3^g\right]\hat{\phi}$$

In equation (46) at θ=0 the term includes the singular part.

(46)

$$\frac{1}{ie}\left(\omega\partial_\mu\omega^\dagger\right)_{singular} = \frac{-1}{e}\left[\frac{(1+cos\theta)}{rsin\theta}\tau_3^g\right]\hat{\phi} \quad (47)$$

This implies that the diagonal part acquire the singular term. Thus in the Abelian gauge, obtained by diagonalizing the field ϕ(x), the gluon field can separated into regular part $\overrightarrow{A^R}$ and the singular part in equation (46) given as.

$$\vec{A} = \overrightarrow{A_a}T_a = \overrightarrow{A_a^R}T_a - \frac{1}{e}\left[\frac{(1+cos\theta)}{rsin\theta}\tau_3^g\right]\hat{\phi} \quad (48)$$

To examine the appearance of the monopole at the origin r = 0, a magnetic flux which penetrates the area inside the closed curve C {r, θ, 0 ≤ ϕ < 2π} is found to be

$$\Phi = \int_C dr.A^{Sing} = \frac{-4\pi}{e}\left(\frac{1+cos\theta}{2}\right)\tau_3^g \quad (49)$$

Which denotes the magnetic flux of monopole situated at the origin, with a endless Dirac string running along the positive z-axis.

$$\Phi^{flux}(\theta = 0) = \frac{-4\pi}{e}\tau_3^g \quad (50)$$

These results may be summarized by saying that topological defects of the Abelian gauge fixing are the sources of magnetic monopoles.

3.2 Appearance of monopoles in decomposition of gluon field

Gluons (in general the non-abelian gauge potential) in field decomposition appear of two different types, the color neutral binding gluons and the colored valance gluons, in gauge independent manner. As a result of this decomposition, the original Yang Mills field theory turns into electrodynamics with magnetic monopoles [7].

In simplest SU (2) QCD field, $\hat{m}$ is an arbitrary local orthonormal basis given by $\hat{m} = \hat{m}_1, \hat{m}_2, \hat{m}_3$. In Cho decomposition the SU (2) Yang Mills field $\overrightarrow{B_\mu}$ decomposes in the following manner.

$$D_\mu\hat{m} = \partial_\mu\hat{m} + g\overrightarrow{B_\mu} \times \hat{m} = 0 \quad (51)$$

Where $\hat{m}$ is the unit vector field that gives the Abelian direction at each space time point and ‘g’ is the Yang Mills coupling constant. Condition (51) restricts the gauge potential $\overrightarrow{B_\mu}$, indicates that the restricted Yang Mills field $\hat{B}_\mu$ is obtained as.

$$\hat{B}_\mu = B_\mu \hat{m} - \frac{1}{g}\hat{m} \times \partial_\mu \hat{m}, B_\mu = \hat{m}.\overrightarrow{B_\mu} \quad (52)$$

The restricted potential $\hat{B}_\mu$ contains two parts: a non topological unrestricted part which is called the electric potential, and the topological restricted part which is related to the magnetic potential. The Abelian part is not restricted by the condition (52) but the magnetic part is completely determined by the magnetic symmetry. Using the restricted potential of equation (52), the field strength $\hat{G}_{\mu\nu}$ is given by.

$$\hat{G}_{\mu\nu} = \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu + g\hat{B}_\mu \times \hat{B}_\nu$$

$$=(F_{\mu\nu} + H_{\mu\nu})\hat{m} \quad (53)$$

Where $F_{\mu\nu}$ is the electric part of field strength and $H_{\mu\nu}$ is the magnetic part of field strength.

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\mu B_\nu$$

$$H_{\mu\nu} = \frac{-1}{g}\hat{m}.(\partial_\mu \hat{m} \times \partial_\nu \hat{m})$$

To introduce a magnetic potential $C_{\mu\nu}$ corresponding to the magnetic field strength $H_{\mu\nu}$(As $F_{\mu\nu}$ is expressed in terms of electric potential B_μ), the magnetic vector $\hat{m}$ can be chosen as.

$$\hat{m} = \begin{bmatrix} sin\alpha cos\beta \\ sin\alpha sin\beta \\ cos\alpha \end{bmatrix} \quad (55)$$

And the magnetic potential expressed as.

$$H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\nu \quad (56)$$

Indeed, $\hat{B}_\mu$ and $\hat{C}_\mu$ are the electric and magnetic contributions of the gluon field. Using equation (54), (55) and (56) the magnetic field strength is given as

$$H_{\mu\nu} = \frac{-1}{g} sin\alpha(\partial_\mu \alpha \partial_\nu \beta - \partial_\nu \alpha \partial_\mu \beta) = (\partial_\mu C_\nu - \partial_\nu C_\nu) \quad (57)$$

A solution of equation (57) is given as.

$$C_\mu = \frac{1}{g}(1 + cos\alpha)\partial_\mu \beta \quad (58)$$

The component of this magnetic potential are.

$$C_r = C_\theta = 0, C_\phi = \frac{1}{g}\frac{(1+\cos\alpha)}{r\sin\alpha} \quad (59)$$

Therefore the magnetic potential has a singularity at α = 0. In space time coordinates in this magnetic potential describe a static Wu-Yang monopole located at the origin with Dirac string along the positive z axis. So the system is Abelianized in the third axis direction in colour space, we can write the singular part of the gluon field as.

$$\hat{B} = \hat{B}\tau_a \rightarrow \hat{B}_{singular} = C\tau_3 = \frac{1}{g}\frac{(1+\cos\alpha)}{r\sin\alpha}\tau_3 \quad (60)$$

Where τ_3 is the diagonal generator of SU (2).Therefore the magnetic charge associated with point like monopole is.

$$g_m = \frac{4\pi}{g}\tau_3 \quad (61)$$

Hence monopole is realised in SU (2) gauge group using Cho decomposition method.

4. Lattice results of magnetic monopole

In this section we will approach to study monopoles within the framework of lattice QCD. Analyzing the interquark potential in lattice QCD provides valuable information about monopole condensation, contributing to the understanding of quark confinement mysteries.

In t'Hooft Abelian projected QCD, potential obtained in lattice QCD can be dissected into the two components, the Abelian component and off-diagonal component [15-20]. Further Abelian component consist monopole part and the photon part. The monopole component associated with Abelian gauge degrees of freedom consist magnetic monopole current only i.e.$k_\mu \neq 0, j_\mu = 0$. On the other hand the photon part only features color electric currents. $j_\mu \neq 0, k_\mu = 0$.

The lattice gauge field $U(s,\mu) = e^{iagA\mu(s)} \in SU(3)$, with lattice spacing 'a', gauge coupling constant 'g' and the gluon field A_μ leads to the Cartan decomposition of the SU(3) group.

$$U^{MA}(s,\mu) = M(s,\mu)u(s,\mu) \in SU(3) \quad (62)$$

The $Q\overline{Q}$potential $V(r)$is obtained with the Wilson loop $W_{r\times t}[U_\mu]$, and its Abelian part $V_{Abel}(r)$is similarly defined $W_{r\times t}[u_\mu]$ by the Abelian Wilson loop . The Abelian link variable projected into$V_{Mon}(r)$ the monopole part $u_{mon}$and photon part u_{ph} leads to the monopole part and the photon part $V_{Ph}(r)$ of the $Q\overline{Q}$ potential. The Monopole part of the $Q\overline{Q}$potential is defined by the monopole link variable as.

$$V_{Mo}(r) = -lim_{T\to\infty}\left[\frac{1}{T} ln\langle W_{R\times T}[u_\mu^{Mo}]\rangle\right]$$

(63)

The lattice result $16^3 \times 32$ lattice at β=5.8, representing the static $Q\bar{Q}$potential V(r) in SU(3) QCD, $V_{Abel}(r)$ in Abelian projected QCD, $V_{Mon}(r)$ in the monopole part, and $V_{ph}(r)$ in the photon part are shown in figure 5 [20].

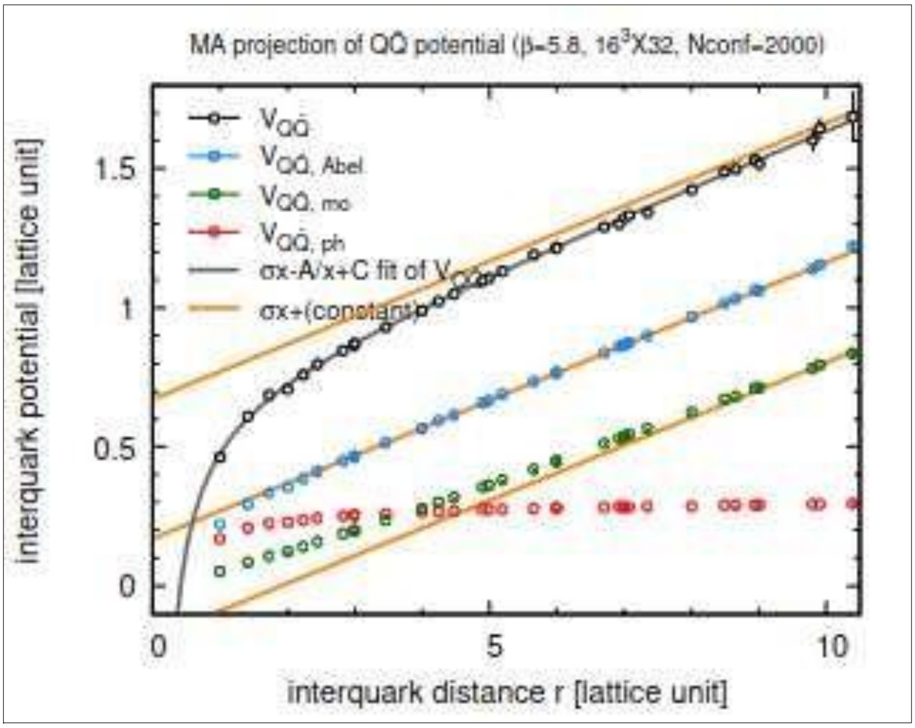

Fig 5: Representation of lattice QCD result for interquark potential V(r) in SU (3) QCD, $V_{Abel}(r)$ in Abelian-projected QCD, $V_{Mon}(r)$ in the monopole part, and $V_{Ph}(r)$ in the photon part for lattice $16^3 \times 32$ at β=5.8

Lattice results reveal a noteworthy observation regarding the SU (3) interquark potential, where the curve of V(r) bears a striking resemblance to the slopes of the curve of $V_{Abel}(r)$as well as $V_{Mon}(r)$. However, it diverges from the profiles of $V_{ph}(r)$. The slope of the interquark potential at large distances represents string tension σ. The slope of monopole part of the potential $\sigma_{mon} = 0.92\sigma$ found in $Q\bar{Q}$in SU (3) QCD [20]. This observation suggests that in the Abelian sector, the monopole component predominantly maintains the confining force, while the photon component exhibits minimal confining force. This observation supports the concept of monopole dominance in driving quark confinement within the system. Consequently, in the maximally Abelian gauge, it becomes evident that color magnetic monopoles encapsulate the quintessence of non-perturbative QCD.

In the context of SU (3) gluodynamics, an alternative decomposition, distinct from equation (62), has been explored [21-24]. This decomposition involves breaking down the nonabelian gauge field into two components: the Abelian field generated by Abelian monopoles and a modified nonabelian field from which the monopoles have been extracted. The abelian projection defined as coset decomposition of the nonabelian lattice gauge field U(s,μ) into the Abelian field u(s, μ) and the coset field C (s, μ) given as.

$$U(s,\mu) = C(s,\mu)u(s,\mu) \quad (64)$$

The Abelian gauge field can in turn be decomposed into the monopole and photon part.

$$u(s,\mu) = u_{mon}(s,\mu)u_{ph}(s,\mu) \quad (65)$$

The modified non Abelian gauge field is defined.

$$\tilde{U}(s,\mu) = C(s,\mu)u_{ph}(s,\mu) \quad (66)$$

$u_{ph}(s,\mu)$ is the Abelian projection of $\tilde{U}(s,\mu)$ and involves no monopoles.

The monopole potential is calculated as.

$$V_{mon}(r) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W_C[u_{mon}(s,\mu)]\rangle \quad (67)$$

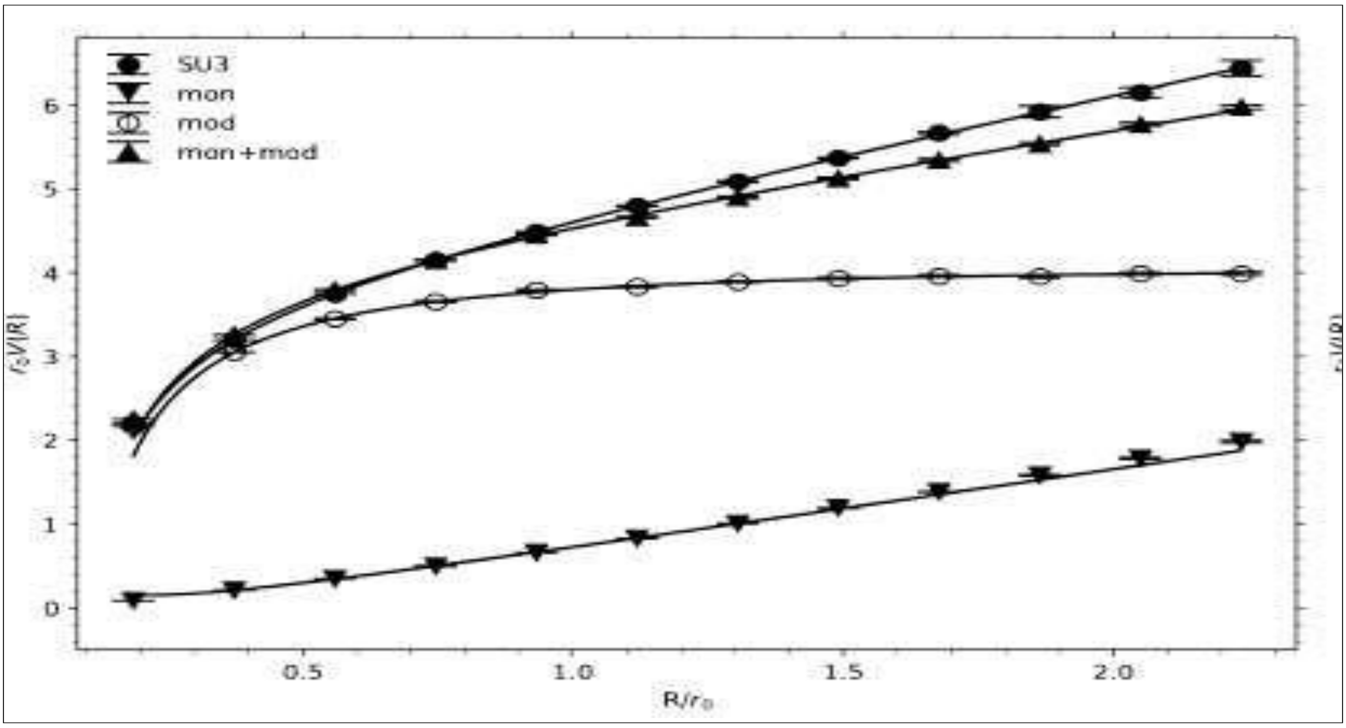

Fig 6: Decomposition of interquark potential into monopole and modified potential for lattice size 24^4 at β= 6.0. [24]

There are few conclusions to be drawn from the decomposition (64) that the monopole static potential also has string tension close to the non-Abelian and are in agreement with conjecture that monopole degrees of freedom are responsible for confinement and suggests that the monopole part $u_{mon}(s,\mu)$is responsible for the classical part of the hadronic string energy and hence advocates the presence of monopole in QCD.

Another widely employed technique, discussed independently in Section 3.2, has been introduced for extracting magnetic monopole degrees of freedom in a gauge-independent manner. In this method, the Abelian projection is defined through magnetic isometry, and the Abelian decomposition is employed to segregate the non-Abelian monopoles. This separation enables the demonstration that the confining potential in QCD arises predominantly from the monopole. Figure 7 illustrates lattice QCD results that confirm monopole dominance in SU (2) QCD [25].

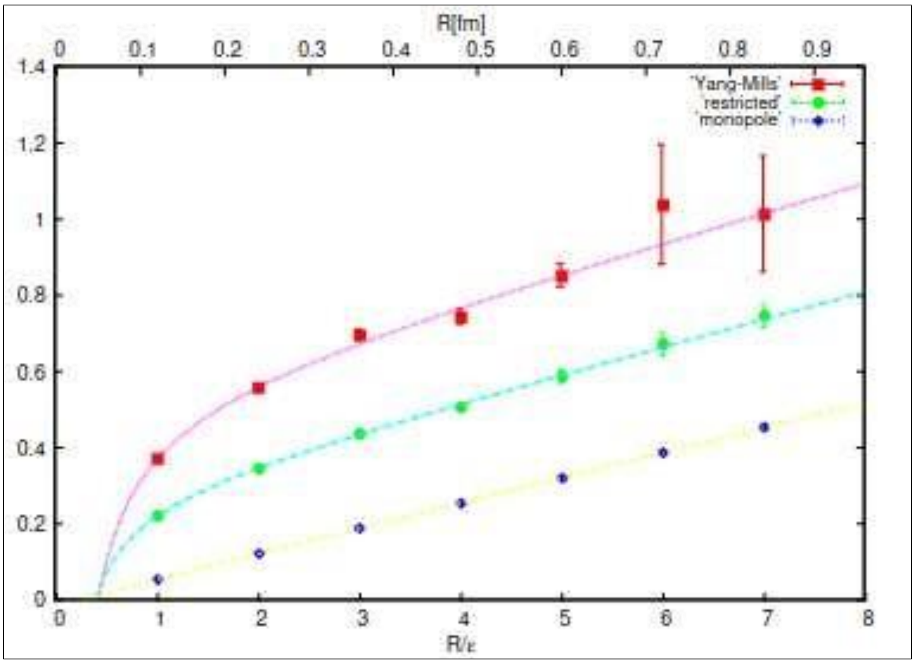

Fig 7: The full SU(2) potential $V_f(R)$and corresponding magnetic-monopole potential,$V_m(R)$ both plotted as functions of R on 16^4lattice at β = 2.4

The lattice calculations demonstrate that the slope of the monopole component of the potential $\sigma_m$reproduces 91% of the slope of the full SU(2) potential $\sigma_f$over the given lattice [26]. This substantiates the magnetic monopole dominance in a gauge-invariant manner. Their work specifically reveals that the confining force in SU(2) QCD originates from the Abelian segment of the potential, with a more precise attribution to the monopole component of the Abelian projection.

5. Discussion and Conclusion

In this paper we have discussed the brief introduction of magnetic monopole given by Dirac and ‘t Hooft and Polyakov. Dual superconducting mechanism has been studied using Abelian projection and Field decomposition method with monopoles as elementary degree of freedom. Further Lattice QCD based on these two methods for the QQ potential in SU(2) and SU(3) QCD explained that the monopole part has potential almost equal to the total confining potential hence favours existence of monopoles in QCD.

6. References

1. Nambu Y. Phys. Rev. D. 1974;10:4262. 't Hooft G. In: High Energy Physics. Bologna: Editorice Compositori; c1975. Mandelstam S. Phys. Rept. 1976;23:245.
2. 't Hooft G. Nucl. Phys. B. 1981;190:455.
3. Ezawa ZF, Iwazaki A. Phys. Rev. D. 1982;25:2681.
4. Dirac PAM. Quantised singularities in the electromagnetic field. Proc. R Soc. Lond Ser. A. 1931;133(821):60-72.
5. 't Hooft G. Magnetic monopoles in unified gauge theories. Nucl. Phys. B. 1974;79(2):276-284.
6. Polyakov AM. Particle Spectrum in the Quantum Field Theory. JETP Lett. 1974;20:194-195.
7. Cho YM. Phys. Rev. D. 1980;21:1080.Cho YM. Phys. Rev. D. 1981;23:2415.
8. Maxwell JC. A dynamical theory of the electromagnetic field. Phil. Trans. R. Soc. Lond. 1865;155:459-512.
9. Levit S. [accessed 11-May-2024]. Available from: https://doi.org/10.48550/arXiv.2304.04247
10. Aharonov Y, Bohm D. Significance of electromagnetic potentials in the quantum theory. Phys. Rev. 1959;115:485-491.
11. Ryder LH. Quantum Field Theory. Cambridge, UK: Cambridge University Press; c1985. p. 403.
12. Wu TT, Yang CN. Dirac’s monopole without strings: Classical Lagrangian theory. Phys. Rev. D. 1976;14:437-445.
13. Yang CN. Generalization of Dirac’s monopole to SU (2) gauge fields. J Math. Phys. 1978;19:320.
14. Ryder LH. Quantum Field Theory. Cambridge, UK: Cambridge University Press; c1985. p. 404.
15. Ichie H, Suganuma H. Dual Higgs Theory for Color Confinement in Quantum Chromodynamics. arXiv:hep-lat/9906005v1.
16. Suganuma H, Sakumichi N. AIP Conference Proceedings. 2016;1701:020016. Sakumichi N, Suganuma H. Phys. Rev. D. 2021;90:111501.
17. Amemiya K, Suganuma H. Phys. Rev. D. 1999;60:114509.
18. Kondo K-I. Phys. Rev. D. 1998;57:7467. Kondo K-I. Phys. Rev. D. 1998;58:105019. Kondo K-I. Phys. Rev. D.
19. 2008;77:085029.
20. Kandpal, Chitra, Singh S, Mishra D. Study on Abelian dominance of Interquark potential in SU (3) Lattice QCD using simulation Techniques. J Mountain Res.; c2019. p. 14.
21. Monopole Dominance of Confinement in SU(3) Lattice QCD; 2019. Available from: arXiv:1812.06827
22. Bornyakov VG, Polikarpov MI, Schierholz G, Suzuki T, Syritsyn SN. Nucl. Phys. Proc. Suppl. 2006;153:25-32.
23. Bali GS, Bornyakov V, Muller-Preussker M, Schilling K. Phys. Rev. D. 1996;54:2863.

24. Bornyakov VG, Kudrov I, Rogalyov RN. Phys. Rev. D. 2022;105:054519.
25. Bornyakov VG, Kudrov IE. JETP Lett. 2023;117:328-331, https://arxiv.org/abs/2301.03076
26. Seikou Kato, Kei-Ichi Kondo, Akihiro Shibata; https://arxiv.org/abs/1407.2808
27. Ito S, Kato S, Kondo K-I, Murakami T, Shibata A, Shinohara T.; https://arxiv.org/pdf/hep-lat/0604016