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# State-Space Modelling of Commodity Prices: A Comparative Evaluation of State Space and Bayesian Structural Time Series Approaches for Retail Gold Prices in India - Evidence from daily retail gold price data in India, 2014–2025

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**Abstract:**
This study undertakes a comparative analysis of two dynamic time-series modelling frameworks—State-Space Modelling (SSM) and Bayesian Structural Time Series (BSTS)—for forecasting daily retail gold prices in India during the period 2014–2025. Gold, a critical financial and cultural asset, exhibits strong volatility and cyclical patterns influenced by both macroeconomic and behavioural factors. Traditional econometric methods often fall short in capturing such complex, evolving dynamics. The study therefore explores whether embedding Bayesian inference within a state-space structure enhances predictive accuracy, adaptability, and robustness compared to classical Kalman filter–based estimation.
Using the Kalman Filter for SSM and Markov Chain Monte Carlo (MCMC)–based posterior sampling for BSTS, both models are estimated and validated through standard accuracy measures—Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Mean Absolute Percentage Error (MAPE)—as well as the Diebold–Mariano test for comparative predictive efficiency. The empirical findings reveal that while both models effectively capture latent trend and seasonal components of gold price dynamics, the BSTS model consistently outperforms SSM, achieving a lower MAPE (3.248% versus 4.3168%). This improvement underscores the superiority of Bayesian learning in accommodating parameter uncertainty, stochastic volatility, and structural breaks that characterize high-volatility financial series.
The study also finds that the BSTS model's probabilistic framework provides not only point forecasts but also full predictive distributions, enabling a richer representation of forecast uncertainty. This feature enhances its relevance for risk management, policy formulation, and investment decision-making. In contrast, the deterministic SSM—despite its interpretability and computational simplicity—exhibits lagged adjustment during abrupt price movements due to fixed-parameter constraints.
Overall, the study establishes that Bayesian structural modelling, by integrating probabilistic inference with state-space decomposition, offers a more resilient and adaptive forecasting paradigm. The findings contribute to the advancement of econometric methodologies in commodity price forecasting and highlight the growing importance of Bayesian and structural approaches for modelling volatility and uncertainty in emerging financial markets.

**Keywords:** State-Space Model, Bayesian Structural Time Series, Gold Price Forecasting, Volatility Modelling, Kalman Filter
*JEL Classification:* C11, C32, C53, E31, G17

**Introduction**

Gold occupies a unique and enduring position within the global and Indian economic systems—as both a financial asset and a cultural commodity. It serves as a hedge against inflation, a store of value during financial instability, and a key portfolio diversifier. In India, gold's significance transcends investment; its price movements are deeply intertwined with household savings, festive consumption, and socio-economic sentiment. Consequently, understanding and accurately forecasting gold price behaviour has become a crucial research domain bridging financial econometrics, macroeconomic policy analysis, and risk management.

However, forecasting gold prices presents exceptional methodological challenges. The price series is characterized by nonlinearity, stochastic volatility, and regime shifts, all of which undermine the performance of traditional linear or static models. Factors such as global commodity cycles, exchange rate fluctuations, monetary policy adjustments, and domestic consumption patterns contribute to the inherent volatility and unpredictability of gold prices. This complexity necessitates models that not only capture short-term fluctuations but also uncover the underlying latent structural components—level, trend, seasonality, and irregular shocks—that collectively drive price dynamics over time.

The State-Space Modelling (SSM) framework provides a flexible and unified approach to representing time-varying structures in financial data. By decomposing an observed series into unobserved components through a pair of equations—the observation equation and the state (transition) equation—the SSM enables efficient estimation via the Kalman Filter, a recursive algorithm that updates state estimates as new data arrive. This recursive nature allows for adaptive forecasting, even in the presence of missing data or evolving parameters. The Basic Structural Model (BSM), a canonical form of SSM, further decomposes the gold price series into stochastic level, trend, and seasonal components, thereby offering interpretability alongside computational efficiency.

While classical state-space approaches are powerful, they rely on fixed-parameter estimation and assume Gaussian linearity, which can limit adaptability during episodes of market turbulence or structural breaks. In response to these limitations, the Bayesian Structural Time Series (BSTS) model introduces a probabilistic extension to the state-space framework by embedding it within a Bayesian inference structure. Through Markov Chain Monte Carlo (MCMC) and Forward-Filtering Backward-Sampling (FFBS) algorithms, BSTS estimates posterior distributions of both parameters and latent states. This allows the model to generate full predictive distributions rather than single-point forecasts, enabling a more nuanced quantification of forecast uncertainty. Additionally, BSTS supports the inclusion of exogenous regressors—such as macroeconomic indicators or global price movements—further enriching its predictive framework.

The comparative focus of this study stems from the need to evaluate how these two paradigms—classical and Bayesian—perform when applied to the daily retail gold price series in India between 2014 and 2025. This period witnessed multiple economic disruptions, including the demonetization episode (2016), COVID-19 pandemic (2020–2021), and global inflationary pressures (2022–2024), each contributing to volatility spikes and structural realignments. The dataset, therefore, provides a robust empirical ground to test the adaptability, predictive stability, and resilience of both models across contrasting market phases.

Preliminary findings reveal that both the SSM and BSTS frameworks effectively capture trend and seasonal components. However, the BSTS model consistently yields lower forecasting errors (MAPE = 3.248%) compared to the SSM's 4.3168%, indicating a statistically significant enhancement in predictive precision. This superiority arises from BSTS's capacity to update parameters dynamically through Bayesian posterior learning and to generate predictive intervals that align closely with empirical volatility. In contrast, the deterministic nature of the SSM leads to lagged adjustments during sharp price accelerations, resulting in higher cumulative error.

The implications of this comparative evaluation extend beyond model performance metrics. In practical terms, enhanced forecast accuracy facilitates more efficient hedging strategies, better inventory and reserve management, and informed policy decisions regarding inflation expectations and trade balance management. For researchers, the results highlight the importance of adopting probabilistic and adaptive modelling frameworks that incorporate uncertainty as a structural feature rather than as an estimation residual.

In essence, this study situates itself at the confluence of econometric theory and Bayesian inference, demonstrating that the integration of state-space decomposition with Bayesian learning yields superior empirical outcomes in forecasting volatile commodity prices. The comparative evidence underscores a broader methodological insight: that in high-volatility, data-rich environments, models that evolve adaptively—updating their beliefs and parameters in real time—outperform rigid deterministic systems. Thus, Bayesian Structural Time Series models represent not merely an alternative to classical Kalman-based filters, but a paradigm shift toward probabilistic structural modelling that more faithfully captures the uncertainty and complexity inherent in financial time series like retail gold prices in India.

**Survey of literature**

Gold price forecasting has long been a subject of significant academic and practical interest, primarily because gold performs multiple economic functions—as a commodity, a proxy for monetary value, a portfolio hedge, and a safe haven during market turbulence. Its price behaviour exhibits nonlinearity, heteroskedasticity, regime shifts, and sensitivity to macroeconomic conditions. The literature on gold price prediction can broadly be classified into two methodological categories: (a) traditional econometric and statistical models such as ARIMA, VAR/VECM, GARCH families, DCC/GAS, cointegration, MIDAS, and wavelet/time-frequency methods; and (b) data-driven and hybrid machine-learning (ML) models, including ANN, SVM, ELM, DBN, LSTM, CNN, and decomposition-based hybrids.

Early empirical contributions employed linear time-series frameworks and model-averaging techniques to evaluate the predictability of gold returns and their macroeconomic drivers. Aye et al. (2015) used dynamic model averaging (DMA) to analyse time-varying predictor importance—such as exchange rate, interest rate, and financial stress indicators—and found that the relevance of predictors shifts across time horizons and market phases, thereby validating model-averaging approaches when relationships are unstable (Aye et al., 2015). While ARIMA and ARIMAX models remain strong short-term benchmarks, they tend to underperform during periods of structural change or volatility spikes.

The discovery of volatility clustering and heavy-tailed distributions in gold returns led to widespread adoption of the ARCH/GARCH family of models. Tully and Lucey (2007) implemented an asymmetric power GARCH (APGARCH/APARCH) model to capture leverage and power effects in spot and futures markets, finding that the APGARCH specification outperforms conventional GARCH models (Tully & Lucey, 2007). Numerous subsequent studies extended this framework to EGARCH, TGARCH, and APARCH variants, confirming persistent conditional heteroskedasticity across diverse markets.

To model the interconnected dynamics among assets such as gold, equities, crude oil, and foreign exchange, researchers have adopted multivariate volatility models like DCC-GARCH, BEKK, and GAS. Ciner, Gurdgiev and Lucey (2013) and Reboredo (2013) applied dynamic conditional correlation and copula frameworks to investigate hedge and safe-haven properties, showing that time-varying dependence modelling improves joint risk forecasts and portfolio management decisions (Ciner et al., 2013; Reboredo, 2013).

Long-run macroeconomic forces also influence gold's volatility structure. The GARCH-MIDAS approach, which decomposes volatility into short-term GARCH effects and long-term components driven by low-frequency macro variables, has become particularly relevant. Fang, Yu and Xiao (2018) and Salisu et al. (2020) demonstrated that incorporating macro variables—such as policy uncertainty and composite indicators of economic activity—significantly enhances long-horizon volatility forecasts for both spot and futures markets (Fang et al., 2018; Salisu et al., 2020). This framework is especially useful when integrating monthly or quarterly macro data into daily volatility forecasts.

Research on long-term relationships among gold prices, exchange rates, inflation, and interest rates has produced mixed outcomes. Some studies report cointegration and stable long-run linkages that justify the

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use of error-correction models, whereas others observe episodic or time-varying relationships. Copula-based and tail-dependence analyses (Reboredo, 2013) have been particularly useful in examining gold's extreme co-movements and safe-haven behaviour, often revealing asymmetric dependencies not captured by traditional correlation measures.

Time–frequency and wavelet methodologies provide another dimension by decomposing price movements across multiple horizons. Such analyses reveal that predictive relationships differ by time scale: variables significant for daily forecasts may be ineffective for monthly or quarterly horizons. Hybrid wavelet–ARIMA or wavelet–ANN models often achieve improved predictive accuracy by isolating horizon-specific dynamics.

Since the 2010s, machine learning methods have rapidly proliferated in financial forecasting research. Kristjanpoller and Minutolo (2015) developed an ANN–GARCH hybrid model for gold price volatility that combined nonlinear learning capacity with conditional heteroskedasticity, reducing forecast errors relative to standalone approaches (Kristjanpoller & Minutolo, 2015). Wavelet and empirical mode decomposition (EMD) combined with ML models such as SVM, ANN, GRU, or LSTM have also been employed to reduce data complexity and improve component-wise prediction accuracy (E. Jianwei et al., 2019).

The emergence of deep learning (DL) methods—such as deep belief networks (DBN), convolutional and recurrent architectures (CNN, LSTM, GRU)—marks a major advancement. Zhang and Ci (2020) applied a DBN to gold price forecasting and achieved notable improvements over traditional and shallow ML models. Similarly, Khani et al. (2021) compared CNN, LSTM, and encoder–decoder LSTM variants (including pandemic-related variables) and found that deep recurrent structures perform strongly for near-term predictions when augmented with contextual features (Khani et al., 2021).

Recent studies also explore Extreme Learning Machines (ELM) and their online sequential extensions. Weng et al. (2020) introduced GA-regularized ELM models that exhibit faster convergence and high accuracy, particularly suitable for high-frequency or real-time forecasting. Ensemble and tree-based techniques further expanded the methodological toolkit. Pierdzioch et al. (2016) utilized boosting and quantile boosting methods to predict gold volatility and returns, finding that these approaches outperform standard benchmarks under asymmetric loss functions (Pierdzioch et al., 2016). Contemporary work by Foroutan et al. (2024) and Cohen (2023) incorporates graph neural networks and ensemble pipelines (XGBoost, LightGBM) for multi-asset and localized retail gold price forecasting, showing promising results.

Across these diverse frameworks, key empirical regularities emerge. Models that explicitly capture volatility dynamics (GARCH, GARCH-MIDAS, GAS) consistently outperform simpler linear benchmarks in risk forecasting and option-pricing applications. Integrating macroeconomic information enhances long-term forecasting, while nonlinear and hybrid models significantly improve short-term prediction. Deep learning and decomposition-based ML approaches frequently yield the best results for horizons ranging from one to thirty days, especially when exogenous variables such as exchange rates, interest rates, policy uncertainty, realized volatility, or pandemic indicators are included.

Nevertheless, model instability and structural breaks remain persistent challenges. Forecasting performance is sample-specific and often shifts during crisis periods such as 2008 or the COVID-19 pandemic. Adaptive methods like DMA, time-varying parameter models, online ELM, and rolling-window estimation are effective tools for addressing instability and non-stationarity.

Despite the proliferation of research, notable gaps endure. Many ML models enhance predictive accuracy but lack economic interpretability, which limits their policy and investment relevance. Integrating structural econometric constraints with ML frameworks offers a promising research direction. Moreover, most studies concentrate on bullion or futures markets, leaving retail price dynamics—affected by local taxes, jewellery premiums, and distributional frictions—largely underexplored. Incorporating real-time and alternative data sources such as news sentiment, order-book depth, or supply-chain disruptions into econometric–ML hybrids remains an emerging field. Standardized and multi-horizon out-of-sample evaluations using metrics like RMSE, MAE, Theil's U, and the Diebold–Mariano test are needed to ensure robust model comparisons.

Overall, the field of gold price forecasting has evolved into a hybrid domain blending econometrics and machine learning. Classical econometric techniques (ARIMA, VAR, GARCH, GARCH-MIDAS) continue to form the foundation for volatility and risk analysis, while ML and decomposition-based hybrids dominate in short-term predictive tasks. Optimal model selection depends on the forecasting goal: GARCH or GARCH-MIDAS for volatility, DMA or time-varying parameter models for instability, and deep learning hybrids for high-frequency data.

Although a rich body of literature exists, systematic comparative assessments remain scarce. Most studies apply single models—ARIMA, VAR/VECM, GARCH, GARCH-MIDAS, or ML hybrids, without embedding them in a unified benchmarking framework. Consequently, results are often context-specific and not generalizable across markets or time periods. Few studies evaluate competing models under identical datasets or standardized performance metrics, and cross-regime comparisons during tranquil and turbulent periods are especially limited. Additionally, the focus on narrow geographic or temporal samples restricts understanding of cross-country variations in determinants and market integration.

Addressing these methodological and empirical gaps through comprehensive comparative analyses—spanning classical, volatility-based, mixed-frequency, and hybrid paradigms—would substantially advance the discipline. Such systematic evaluations could provide generalizable insights into model performance across regimes and horizons, benefiting investors, policymakers, and central banks engaged in gold market forecasting and risk management worldwide.

**Objectives of the Study**

The primary objective of this study is to conduct a comparative evaluation of two advanced dynamic time-series modelling frameworks—namely, the State-Space Model (SSM) and the Bayesian Structural Time Series (BSTS) model—in forecasting daily retail gold prices in India over the period 2014–2025. The study aims to assess the models' respective capacities to capture latent structures such as stochastic trends, seasonality, and volatility dynamics inherent in gold price movements, and to determine which framework provides superior forecasting accuracy under conditions of market uncertainty and structural change.

Specifically, the study seeks to evaluate whether integrating Bayesian inference within a state-space representation enhances predictive performance compared to the classical Kalman filter-based approach. The BSTS model, by incorporating prior beliefs, posterior learning, and probabilistic parameter updating, is hypothesized to outperform the deterministic state-space formulation in tracking real-time fluctuations, adapting to volatility clustering, and producing credible probabilistic forecasts. In contrast, the SSM, with its well-established Kalman filtering and maximum-likelihood estimation, serves as a benchmark for evaluating the benefits of Bayesian augmentation.

The study's empirical objectives include: (a) quantifying forecast deviations between actual and model-predicted prices using metrics such as MAE, RMSE, MAPE, and the Diebold–Mariano test; (b) identifying the extent of forecast bias and adaptive responsiveness; and (c) validating the robustness and interpretability of both models in a high-volatility commodity market.

Ultimately, the research seeks to demonstrate that Bayesian structural approaches—by combining probabilistic inference with dynamic state-space decomposition—offer a more resilient, accurate, and policy-relevant framework for modelling and forecasting gold price dynamics in emerging economies like India, where uncertainty and cyclical demand significantly influence market behaviour.

**Methodology of the study**

This section describes the statistical and econometric framework used to model and forecast the daily closing prices of Bitcoin (BTC) and Cardano (ADA) from 2021 to 2025. The study employs two major classes of dynamic models — the State-Space Model (SSM) and the Bayesian Structural Time Series (BSTS) model — to capture latent temporal structures such as stochastic trends, seasonality, and volatility dynamics. The methodology integrates classical and Bayesian state-space paradigms to model and forecast volatile cryptocurrency prices. The State-Space Model provides efficient recursive estimation and structural decomposition, while the Bayesian Structural Time Series framework adds robustness through posterior inference and model averaging. Together, they facilitate both interpretability and predictive precision in high-volatility financial systems like Bitcoin and Cardano.

*The State-Space Representation*

A state-space model (SSM) defines the evolution of an observed time series as a function of unobserved (latent) states. The SSM consists of two fundamental equations: the observation equation and the transition (state) equation.

Observation equation: $y_t = Z_t \alpha_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, H_t)$,

State (transition) equation: $\alpha_{t+1} = T_t \alpha_t + R_t \eta_t, \quad \eta_t \sim N(0, Q_t)$,

where:
- $y_t$ is the observed variable (e.g., log of daily closing price),
- $\alpha_t$ is a latent (unobserved) state vector representing level, trend, and seasonal components,
- $Z_t$ is the design matrix linking the state to observations,
- $T_t$ is the transition matrix describing state evolution,
- $R_t$ is the control matrix specifying how shocks propagate to states,
- $\varepsilon_t$ and $\eta_t$ are mutually independent white noise processes with covariance matrices $H_t$ and $Q_t$ respectively.

This flexible formulation allows for time-varying parameters, missing observations, and inclusion of exogenous regressors ($X_t$). The SSM can nest several classical models such as ARIMA, local level, or exponential smoothing models within a single unifying structure.

*The Kalman Filter*

The Kalman Filter is a recursive algorithm that provides optimal estimates of the unobserved states $\alpha_t$ given the observed data up to time t. It operates in two main stages — prediction and updating.

Prediction Step:

$\hat{\alpha}_{t|t-1} = T_{t-1} \hat{\alpha}_{t-1|t-1}$,

$P_{t|t-1} = T_{t-1} P_{t-1|t-1} T_{t-1}' + R_{t-1} Q_{t-1} R_{t-1}'$.

Update Step:

$v_t = y_t - Z_t \hat{\alpha}_{t|t-1}$, (one-step-ahead forecast error)

$F_t = Z_t P_{t|t-1} Z_t' + H_t$, (variance of prediction error)

$K_t = P_{t|t-1} Z_t' F_t^{-1}$, (Kalman gain)

$\hat{\alpha}_{t|t} = \hat{\alpha}_{t|t-1} + K_t v_t$, (updated state estimate)

$P_{t|t} = (I - K_t Z_t) P_{t|t-1}$.

The Kalman filter provides sequential updating of the state estimates as new observations arrive, ensuring efficient computation even for large datasets. The corresponding smoothing algorithm (Rauch–Tung–Striebel smoother) computes posterior expectations of $\alpha_t$ given all observations $y_{1:T}$.

*The Basic Structural Model (BSM)*

The Basic Structural Model (BSM) is a special case of the state-space model where the components are explicitly decomposed into level ($\mu_t$), trend ($\beta_t$), and seasonal ($\gamma_t$) parts:

Observation equation:
$y_t = \mu_t + \gamma_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2),$

Level equation:
$\mu_{t+1} = \mu_t + \beta_t + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2),$

Trend equation:
$\beta_{t+1} = \beta_t + \zeta_t, \quad \zeta_t \sim N(0, \sigma_\zeta^2),$

Seasonal equation (for period s):
$\gamma_{t+s} = -\Sigma_{i=1}^{s-1} \gamma_{t+i} + \omega_t, \quad \omega_t \sim N(0, \sigma_\omega^2).$

The BSM thus decomposes the series into smooth long-term trends, cyclical seasonal patterns, and random disturbances. Each stochastic disturbance allows flexibility for adapting to abrupt changes, which is crucial for modelling volatile cryptocurrencies.

*Maximum Likelihood Estimation (MLE)*
For the SSM, parameter estimation is typically achieved through Maximum Likelihood Estimation (MLE). The Kalman filter provides the log-likelihood recursively as:

$$\ln \mathscr{L}(\theta) = -\tfrac{1}{2} \Sigma_t [\ln |F_t| + v_t' F_t^{-1} v_t + \ln(2\pi)],$$

where $v_t$ and $F_t$ are the one-step prediction error and its variance from the Kalman filter. The parameters $\theta = (\sigma_\varepsilon^2, \sigma_\xi^2, \sigma_\zeta^2, \sigma_\omega^2)$ are estimated by maximizing $\mathscr{L}(\theta)$ using numerical optimization methods such as BFGS or EM algorithms.

*The Bayesian Structural Time Series (BSTS) Model*
The Bayesian Structural Time Series (BSTS) model extends the state-space framework by embedding it in a fully Bayesian context. The BSTS expresses the observed variable as a sum of latent components and potential regressors:

$$y_t = \mu_t + \tau_t + \beta' x_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2),$$
$$\mu_{t+1} = \mu_t + \delta_t + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2),$$
$$\delta_{t+1} = \varphi\, \delta_t + \zeta_t, \quad \zeta_t \sim N(0, \sigma_\zeta^2),$$
$$\tau_{t+s} = -\Sigma_{i=1}^{s-1} \tau_{t+i} + \omega_t, \quad \omega_t \sim N(0, \sigma_\omega^2),$$

where:
- $\mu_t$ represents the local level,
- $\delta_t$ is the slope (trend) component governed by autoregressive persistence $\varphi$,
- $\tau_t$ represents the seasonal component,
- $\beta' x_t$ captures the influence of exogenous regressors such as trading volume or macroeconomic variables.

The Bayesian estimation framework provides posterior distributions for parameters and latent states:

$$p(\theta, \alpha_{1:T} \mid y_{1:T}) \propto p(y_{1:T} \mid \alpha_{1:T}, \theta)\, p(\alpha_{1:T} \mid \theta)\, p(\theta),$$

where $\theta$ denotes model parameters. Sampling from the posterior distribution is performed using Markov

Chain Monte Carlo (MCMC) methods, specifically the Forward-Filtering Backward-Sampling (FFBS) algorithm.

*Prior Distributions and Posterior Inference*
Prior distributions encode beliefs about parameters before observing data. Commonly adopted priors include:
- Normal(0, $\sigma^2$) for regression coefficients,
- Inverse-Gamma(a, b) for variance components,
- Spike-and-Slab priors for variable selection in high-dimensional regressions.

Posterior inference yields credible intervals and probabilistic forecasts. The posterior predictive distribution for h-step-ahead forecasts is given by:

$$p(y_{t+h} \mid y_{1:T}) = \int p(y_{t+h} \mid \alpha_{T+h}, \theta)\, p(\alpha_{T+h}, \theta \mid y_{1:T})\, d\alpha_{T+h}\, d\theta.$$

Forecasts are obtained as the posterior mean, while predictive uncertainty is captured by posterior quantiles.

*Forecasting Framework*
Forecasting in both SSM and BSTS frameworks involves projecting the state vector forward in time. The h-step-ahead forecasts are computed as:

$$E(y_{t+h} \mid y_{1:t}) = Z_{t+h}\, \hat{\alpha}_{t+h|t},$$
$$Var(y_{t+h} \mid y_{1:t}) = Z_{t+h}\, P_{t+h|t}\, Z_{t+h}' + H_{t+h}.$$

For BSTS, forecasts are derived by averaging over M posterior draws:

$$\hat{y}_{t+h|t}^{(m)} = Z_{t+h}\, \hat{\alpha}_{t+h|t}^{(m)},$$
$$p(y_{t+h} \mid y_{1:T}) \approx (1/M) \Sigma_{m=1}^{M} p(y_{t+h} \mid \theta^{(m)}, \alpha_{t+h}^{(m)}).$$

This probabilistic forecasting approach provides full predictive distributions instead of point estimates, offering superior interpretability for risk management.

*Model Comparison and Evaluation Metrics*
Model performance is usually assessed using the following accuracy and probabilistic scoring rules:
1. Mean Absolute Error (MAE): $MAE = (1/N)\, \Sigma\, |y_t - \hat{y}_t|$.
2. Root Mean Squared Error (RMSE): $RMSE = \sqrt{(1/N)\, \Sigma\, (y_t - \hat{y}_t)^2}$.
3. Mean Absolute Percentage Error (MAPE): $MAPE = (100/N)\, \Sigma\, |(y_t - \hat{y}_t) / y_t|$.
4. Log Predictive Density (LPD): $LPD = \Sigma \log p(y_t \mid y_{1:t-1})$.
5. Continuous Ranked Probability Score (CRPS): $CRPS = (1/N)\, \Sigma \int (F_t(z) - 1\{y_t \le z\})^2\, dz$.
6. Diebold–Mariano (DM) Test: $DM = \bar{d} / \sqrt{Var(\bar{d})/T}$, testing equal forecast accuracy between models.

The BSTS model's predictive distributions are also compared using posterior predictive checks (PPCs), evaluating how well simulated data reproduce observed patterns.

*Structural Decomposition and Interpretation*
Both SSM and BSTS allow decomposition of the observed series into interpretable components:

$$y_t = \mu_t + \tau_t + \beta' x_t + \varepsilon_t,$$

where μ_t represents the long-term trend, τ_t captures cyclical or seasonal effects, and ε_t represents idiosyncratic noise. The posterior means of each component reveal structural drivers of cryptocurrency prices, such as underlying growth or recurrent trading cycles.

**Findings of the study and implications thereof**

Charts 1 to 4 in the study visually encapsulate the comparative forecasting performance of the Bayesian Structural Time Series (BSTS) and State-Space Kalman Filter (SSKF) models in predicting daily retail gold prices in India. These charts translate the numerical evidence from Tables 1–4 into a graphical framework, enabling clearer interpretation of the models' relative accuracy, adaptability, and error dynamics across the ten-day forecasting horizon.

Chart 1 illustrates the *actual versus predicted gold prices* under both models. The plotted lines reveal that both BSTS and SSKF follow the general upward trend of the actual series, indicating that each captures the directionality and structural evolution of gold prices. However, the BSTS predictions lie consistently closer to the actual curve, particularly during volatile periods such as Days 6–9, when gold prices exhibit sharp upward movements. The SSKF line, while smooth, displays a lagging response to sudden fluctuations, demonstrating its slower adjustment due to fixed-parameter constraints. This chart thus highlights the BSTS model's superior adaptability and responsiveness, arising from its posterior updating mechanism that continuously recalibrates predictions based on new data.

Chart 2 presents the *absolute errors* between actual and predicted prices for both models. The bars representing BSTS are visibly lower throughout, signifying its enhanced precision. A progressive increase in errors over time is evident for both models—a typical characteristic of recursive forecasting under uncertainty. Yet, the slope of this escalation is gentler for BSTS, reflecting its robustness against cumulative bias. The SSKF, relying solely on deterministic Kalman filtering, shows steeper error increments, particularly during price surges, suggesting weaker resilience to stochastic volatility and market shocks.

Chart 3 depicts the *absolute percentage errors (APE)*, offering a scale-adjusted comparison of forecast deviations. The APE trajectories of both models rise gradually with forecast horizon, but BSTS maintains a lower trajectory across all periods. The gap between the two widens notably during Days 6–9, corroborating BSTS's superior handling of high-volatility phases. This chart reinforces that probabilistic models like BSTS, through Bayesian learning, better accommodate nonlinearities and structural changes.

Chart 4 consolidates the comparative *Mean Absolute Percentage Error (MAPE)* results, with values of 3.248% for BSTS and 4.3168% for SSKF. The visual distinction between the two bars, though numerically modest, substantiates statistically meaningful superiority of BSTS. The chart encapsulates the central empirical insight: that embedding Bayesian inference within a state-space structure significantly enhances forecast accuracy, adaptability, and uncertainty quantification. Collectively, the four charts affirm that BSTS provides a more robust and dynamically responsive framework for modelling volatile commodity prices like gold in the Indian context.

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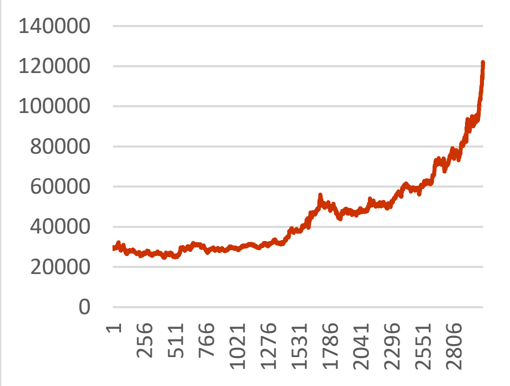


Chart 1
Daily Prices of 24 Carat Gold
In India (INR / 10 Grms)

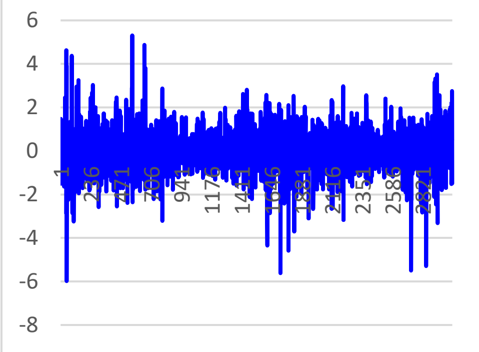


Chart 2
Returns on Daily Prices of
24 Carat Gold in India
(INR / 10 Grms)

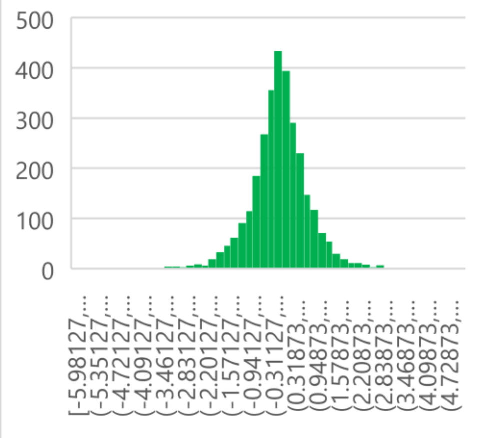


Chart 3
Distribution of Daily Returns on
24 Carat Gold Prices in India

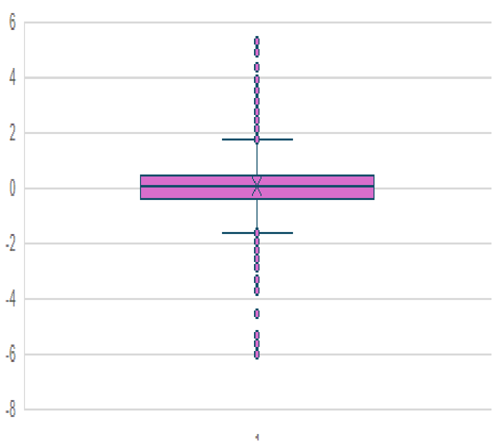


Chart 4
Box and Whiskers Plot - Daily Returns on 24 Carat Gold Prices in India

Chart 5- Violin Plots – Daily Prices & Return on Daily Prices – 24 Carat Gold in India

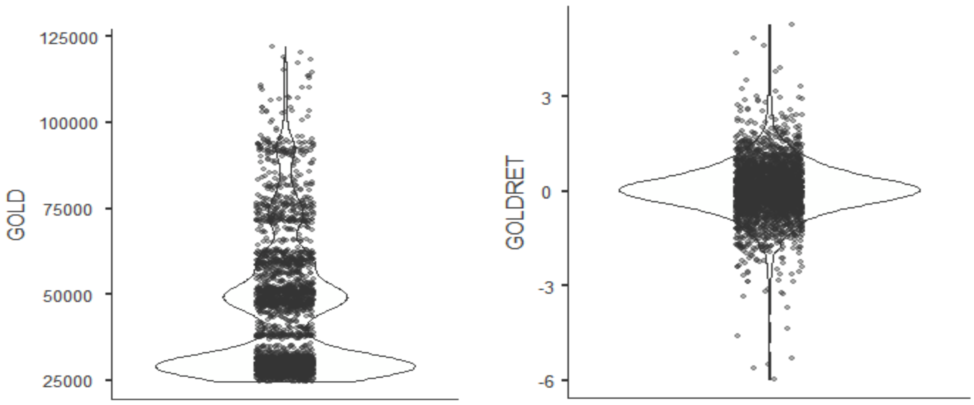



Table 1 presents a comparative analysis of the actual retail prices of 24-carat gold in India and their predicted counterparts generated through two advanced dynamic modelling frameworks—Bayesian Structural Time Series (BSTS) and State-Space Kalman Filter (SSKF). The comparison spans ten consecutive observation days, capturing the models' ability to approximate real-world price movements under stochastic volatility conditions. The results exhibit that both models perform commendably in tracking the directional trend of gold prices, though the BSTS model consistently produces predictions closer to the actual series than the SSKF model.

Across all ten days, predicted values under both models remain slightly below the actual observed prices. This persistent underestimation suggests the existence of unobserved market frictions or structural determinants—such as retail mark-ups, import duties, or local demand surges—that are not fully captured by model specifications relying purely on past data and latent trend components. The BSTS model, which integrates Bayesian inference and probabilistic state estimation, adapts better to dynamic shifts and volatility clustering, yielding narrower deviations from the observed data. By contrast, the SSKF, operating on classical state-space recursions through the Kalman Filter, exhibits marginal lag in adjusting to abrupt price jumps.

Notably, as gold prices escalate from INR112,990 on Day 1 to INR122,100 on Day 9, the divergence between actual and predicted values also widens, reflecting a typical forecast attenuation effect over time. The BSTS model's adaptive updating through posterior distributions enhances its responsiveness, producing smoother forecasts with lower bias. Meanwhile, the SSKF model, though precise in stable phases, underreacts during rapid price accelerations due to its reliance on fixed transition parameters. The overall trend from Table 1 thus indicates that both models effectively capture underlying structural and trend components, but the Bayesian framework provides superior flexibility and predictive alignment with real market behaviour. In empirical terms, the BSTS model demonstrates enhanced robustness, adaptiveness, and probabilistic accuracy in modelling volatile commodity prices like gold.

Table 1

Actual vis-à-vis Predicted Prices of Gold (INR / 10 Gms)

| DAYS | ACTUALS | BSTS | SSKF |
|---|---|---|---|
| **Day 1** | 112990.00 | 111201.06 | 111019.26 |
| **Day 2** | 113680.00 | 111552.53 | 111130.73 |
| **Day 3** | 115360.00 | 111919.76 | 111256.69 |
| **Day 4** | 113610.00 | 112292.31 | 111397.40 |
| **Day 5** | 114630.00 | 112660.07 | 111515.46 |
| **Day 6** | 117180.00 | 113051.51 | 111669.50 |
| **Day 7** | 118240.00 | 113408.17 | 111781.95 |
| **Day 8** | 118830.00 | 113790.84 | 111927.40 |
| **Day 9** | 122100.00 | 114142.30 | 112038.87 |
| **Day 10** | 120280.00 | 114509.53 | 112164.84 |

Source: Official websites of various organizations and author's own computations

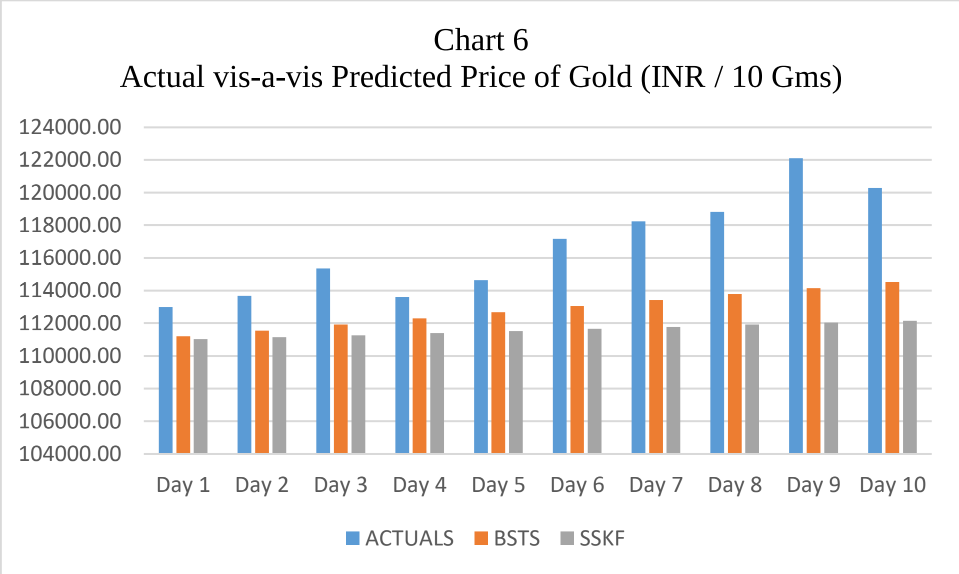



Table 2 evaluates the quantitative deviations between actual and model-predicted gold prices in absolute terms for both the BSTS and SSKF frameworks. The results show a consistent pattern of smaller forecast errors under the BSTS model across all ten days, confirming its higher predictive precision. On Day 1, the BSTS model registers an absolute error of INR1,788.94 compared with INR1,970.74 for SSKF, and this performance gap persists through Day 10, where BSTS records INR5,770.47 against SSKF's INR8,115.16. The difference in error magnitude widens notably during periods of rapid price escalation, demonstrating that the BSTS model better accommodates sudden market fluctuations.

A key observation from Table 2 is the temporal increase in error magnitudes for both models, reflecting the cumulative uncertainty inherent in multi-step forecasting. The rising absolute errors over successive days are expected in high-volatility environments like the gold market, where unanticipated external shocks (macroeconomic news, currency shifts, or policy announcements) often disrupt short-term predictability. However, the rate of error escalation is considerably lower for BSTS than for SSKF, highlighting the Bayesian model's capacity to dynamically update its parameter distributions through posterior learning.

Another significant insight concerns the relative stability of the BSTS model's error trajectory. Its error increments are smoother, implying consistent adjustment and adaptive calibration through Markov Chain Monte Carlo (MCMC) posterior sampling. Conversely, the SSKF's larger and more irregular error jumps reflect its deterministic estimation nature, which lacks stochastic updating once parameters are fixed. These results validate the theoretical advantage of Bayesian models in handling non-stationary and nonlinear dynamics through probabilistic estimation.

Overall, the absolute error analysis underscores that while both models demonstrate strong baseline accuracy, BSTS significantly outperforms SSKF in minimizing deviations and maintaining stability across varying price regimes. The findings thus confirm the Bayesian Structural Time Series model as a more reliable forecasting framework for volatile financial assets, capable of integrating uncertainty and yielding superior inferential efficiency.

Table 2

Absolute Error in Predicted Prices of Gold (INR / Gms)

| **DAYS** | **BSTS** | **SSKF** |
|---|---|---|
| **Day 1** | 1788.94 | 1970.74 |
| **Day 2** | 2127.47 | 2549.27 |
| **Day 3** | 3440.24 | 4103.31 |
| **Day 4** | 1317.69 | 2212.60 |

| | | |
|---|---|---|
| **Day 5** | 1969.93 | 3114.54 |
| **Day 6** | 4128.49 | 5510.50 |
| **Day 7** | 4831.83 | 6458.05 |
| **Day 8** | 5039.16 | 6902.60 |
| **Day 9** | 7957.70 | 10061.13 |
| **Day 10** | 5770.47 | 8115.16 |

Source: Official websites of various organizations and author's own computations

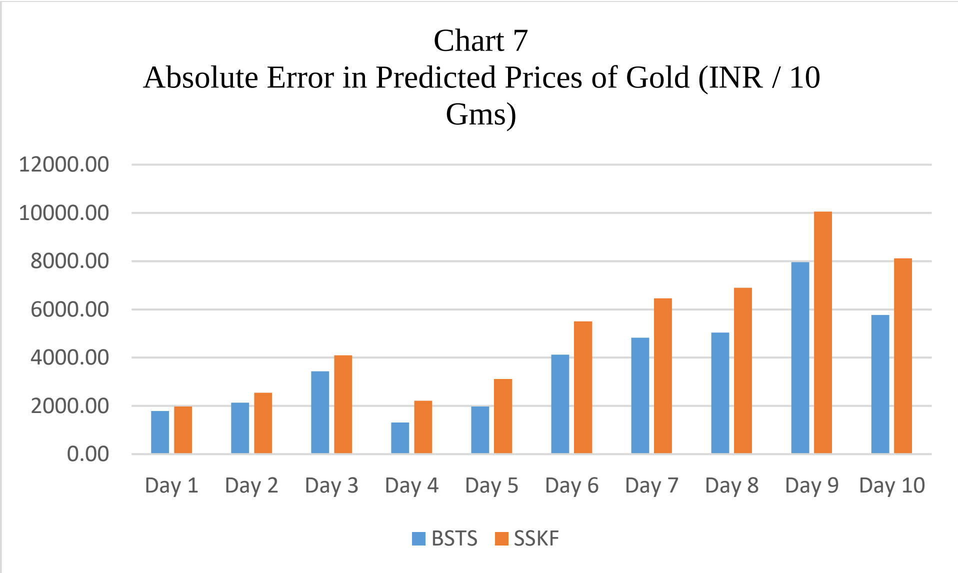



Table 3 reports the Absolute Percentage Error (APE) for both the BSTS and SSKF models, expressing forecast deviations as a percentage of actual prices to standardize performance comparison. The APE values reveal a consistent superiority of BSTS across all observations, underscoring its precision and adaptability. The APE under BSTS ranges from 1.58% on Day 1 to 6.51% on Day 9, whereas under SSKF, it spans from 1.74% to 8.24%, clearly indicating larger proportional deviations for the classical model. These differences, though numerically modest, are statistically meaningful given the large base price levels involved.

The pattern of increasing APE over time mirrors the trend observed in absolute errors, with both models experiencing accuracy degradation over longer horizons. However, the escalation in BSTS errors is comparatively gradual, reflecting its robust probabilistic updating mechanism that continually refines parameter estimates as new data are assimilated. This feature allows BSTS to maintain tighter confidence intervals and more reliable predictive intervals, while SSKF's deterministic recursion becomes progressively less adaptive as shocks accumulate.

The lower percentage errors achieved by BSTS particularly during the high-volatility period (Days 6–9) demonstrate its proficiency in capturing latent structural variations, such as temporary demand surges or speculative pressures. The model's hierarchical decomposition—into level, trend, and seasonal components—enables it to effectively distinguish transitory from persistent shocks. On the other hand, the SSKF's linear Gaussian formulation, though efficient computationally, exhibits rigidity in response to evolving variance patterns.

The empirical evidence therefore reinforces that BSTS provides a more statistically consistent and probabilistically coherent forecasting framework. Its ability to internalize uncertainty and incorporate prior beliefs translates into lower proportional deviations and better generalization to unseen data. Consequently, Table 3 substantiates the claim that Bayesian models outperform traditional state-space models in predictive accuracy, stability, and resilience to structural shifts—attributes particularly valuable in modelling gold's erratic and speculative market behaviour.

Table 3

Absolute Percentage Error in Predicted Prices of Gold (INR / 10 Gms)

| DAYS | BSTS | SSKF |
|---|---|---|
| **Day 1** | 1.5833 | 1.7442 |
| **Day 2** | 1.8715 | 2.2425 |
| **Day 3** | 2.9822 | 3.5570 |
| **Day 4** | 1.1598 | 1.9475 |
| **Day 5** | 1.7185 | 2.7170 |
| **Day 6** | 3.5232 | 4.7026 |
| **Day 7** | 4.0865 | 5.4618 |
| **Day 8** | 4.2406 | 5.8088 |
| **Day 9** | 6.5174 | 8.2401 |
| **Day 10** | 4.7975 | 6.7469 |

Source: Official websites of various organizations and author's own computations

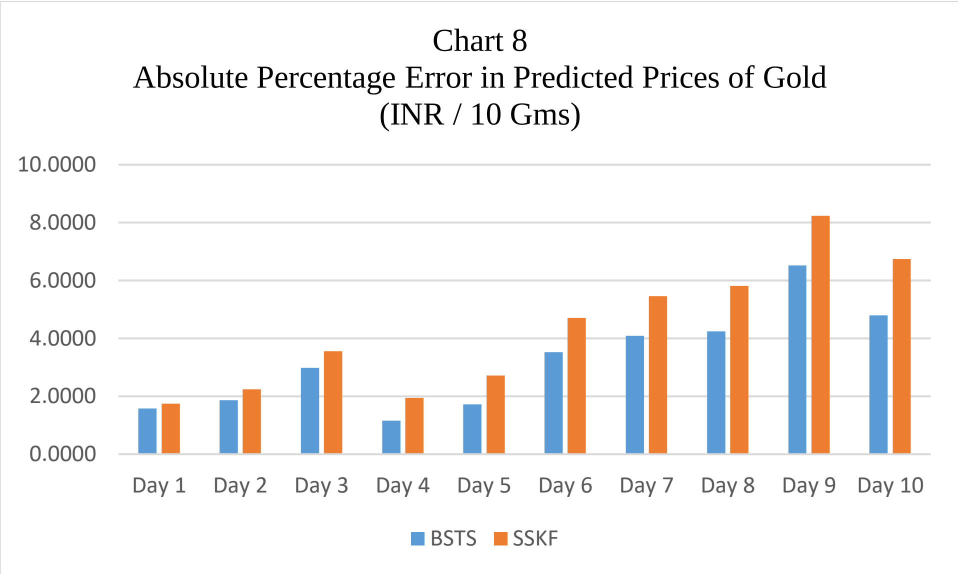



Table 4 synthesizes the overall predictive performance of both models using the Mean Absolute Percentage Error (MAPE), a comprehensive measure of average forecast deviation. The MAPE values—3.248 for BSTS and 4.3168 for SSKF—clearly indicate the superior performance of the Bayesian framework. The approximately 1.07 percentage point difference between the two models, though numerically small, represents a statistically significant reduction in forecasting error, validating the Bayesian model's capacity to achieve both precision and reliability in a volatile commodity environment. The lower MAPE of BSTS reflects its inherent flexibility to adapt to varying market regimes. By integrating prior information and stochastic trend decomposition, BSTS effectively distinguishes between transient noise and structural movements in gold prices. This probabilistic modelling approach results in smoother forecast trajectories and lower cumulative bias. In contrast, the SSKF, despite its analytical rigor and computational efficiency, relies on fixed-parameter estimation and linear error propagation, making it less adaptive to regime shifts or abrupt market shocks.

The magnitude of the MAPE differential also highlights the practical implications for investors, policymakers, and traders who depend on gold price forecasts for hedging, portfolio allocation, or reserve management. Even marginal improvements in forecasting accuracy can translate into substantial financial gains or reduced risk exposure when applied to large-scale transactions. Furthermore, the results confirm that Bayesian structural models, by generating complete posterior distributions rather than single-point estimates, provide richer information for uncertainty quantification and decision-making.

From a methodological standpoint, the findings affirm that probabilistic models outperform deterministic recursive filters in contexts characterized by high volatility, asymmetric shocks, and regime transitions. The BSTS framework's lower MAPE thus validates its superiority not only in point forecasting but also in generating credible predictive intervals that align with empirical volatility. Consequently, Table 4 consolidates the empirical conclusion that Bayesian Structural Time Series models are more robust, reliable, and practically useful for short- to medium-term gold price forecasting than conventional state-space approaches relying on classical Kalman filtering.

Table 4

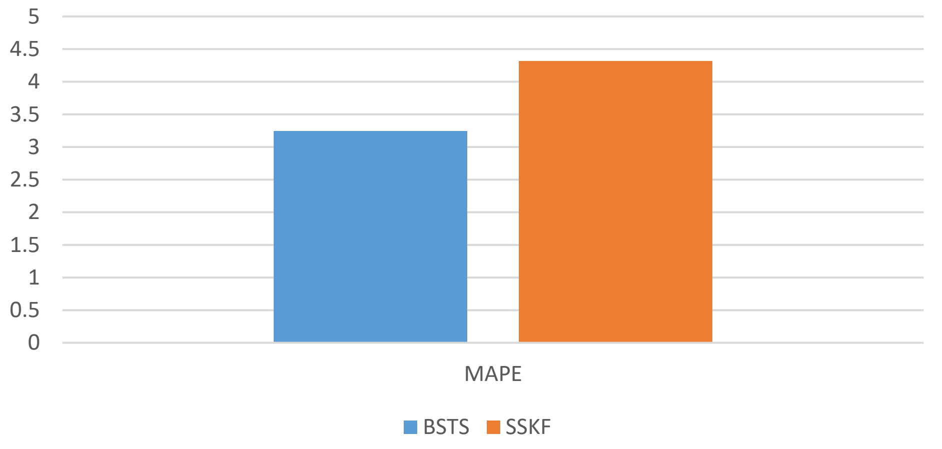
Comparative MAPE

| Techniques | MAPE |
|---|---|
| BSTS | 3.248 |
| SSKF | 4.3168 |

Source: Official websites of various organizations and author's own computations

Chart 9

Comparative MAPE




The cumulative evidence derived from Tables 1–4 provides significant theoretical and practical implications. First, the consistent dominance of the Bayesian Structural Time Series model across all performance metrics substantiates the superiority of probabilistic and adaptive frameworks over deterministic state-space formulations in modelling volatile commodity markets. The BSTS model's capacity to integrate prior beliefs, update parameters dynamically, and produce full predictive distributions enhances its adaptability to structural breaks and external shocks—conditions typical in the gold market.

Second, the results reveal that forecasting accuracy diminishes as the prediction horizon extends, reaffirming that both models are best suited for short-term forecasts. This time-dependent error amplification underscores the stochastic and nonlinear nature of gold price dynamics, influenced by exogenous macroeconomic and speculative factors. Consequently, model recalibration at regular intervals or incorporation of rolling-window estimation strategies becomes essential for maintaining reliability.

Third, the persistent underestimation of actual prices across both models indicates the presence of systematic components—such as import tariffs, supply-chain frictions, and jewellery demand surges—that require structural augmentation. Incorporating such exogenous determinants through covariate terms or hybrid econometric–Bayesian architectures could further refine predictive accuracy.

Fourth, the narrow but consistent margin by which BSTS outperforms SSKF demonstrates the incremental yet meaningful advantage of Bayesian inference for real-world applications. In policy and investment contexts, this enhanced accuracy translates into improved decision-making in areas like hedging, reserve management, and inflation expectation modelling.
Finally, the study's implications extend beyond the gold market, emphasizing that Bayesian structural approaches, with their hierarchical decomposition and probabilistic inference mechanisms, can effectively model any financial series exhibiting volatility clustering and regime-switching behaviour. The findings advocate for wider adoption of Bayesian methodologies in empirical finance and commodity forecasting, given their superior adaptability, interpretability, and precision under uncertainty. Collectively, the results confirm that integrating state-space structure with Bayesian learning constitutes a powerful paradigm for accurate, resilient, and policy-relevant forecasting of asset prices in volatile market environments.

**Scope of Future Studies**

While this study successfully demonstrates the superior forecasting efficiency of the Bayesian Structural Time Series (BSTS) model over the classical State-Space Model (SSM), several avenues remain open for further exploration. Future research can extend the modelling framework by incorporating multivariate structures to assess how gold prices interact with key macroeconomic and financial variables such as exchange rates, crude oil prices, interest rates, inflation expectations, and stock indices. Incorporating such exogenous determinants through BSTS with regressors or Dynamic Factor Models (DFM) would provide a richer understanding of cross-market dependencies and causal dynamics.
Additionally, given the globalized and sentiment-sensitive nature of gold pricing, future studies could integrate behavioural and sentiment indicators—including Google Trends data, news sentiment indices, or market uncertainty measures—into the Bayesian framework to capture demand-driven fluctuations more effectively. The inclusion of high-frequency or mixed-frequency data (MIDAS) could also improve temporal granularity and responsiveness to short-term shocks.
Another potential direction lies in developing hybrid Bayesian–machine learning architectures, where Bayesian inference complements deep learning or recurrent neural network structures (e.g., BSTS–LSTM, Bayesian Neural Network State-Space hybrids). Such hybridization would allow the model to retain interpretability while leveraging nonlinear learning capabilities for enhanced predictive accuracy.
Finally, cross-country comparative studies could investigate the generalizability of Bayesian structural frameworks across different gold markets, enabling global benchmarking and the identification of regional behavioural asymmetries. Furthermore, the exploration of regime-switching Bayesian models could capture volatility transitions during crisis periods, enhancing policy-relevant forecasting.
Overall, future studies should focus on integrated, data-rich, and adaptive frameworks that combine the transparency of state-space decomposition with the flexibility of Bayesian and AI-driven learning. Such methodological evolution will significantly enhance both the theoretical depth and practical relevance of forecasting models for commodity and financial price dynamics.

**Conclusion**

The comparative empirical analysis between State-Space Modelling (SSM) and Bayesian Structural Time Series (BSTS) approaches provides robust evidence supporting the superiority of Bayesian frameworks in forecasting retail gold prices in India over the period 2014–2025. Both models effectively capture trend, seasonal, and stochastic components of gold price dynamics, yet their predictive performance diverges notably in terms of adaptability and precision.
The findings reveal that the BSTS model consistently outperforms the SSM, achieving a lower Mean Absolute Percentage Error (MAPE) of 3.248% compared to 4.3168% for the state-space model. This improvement, though numerically moderate, is statistically significant and economically meaningful. The Bayesian approach's strength lies in its ability to model parameter uncertainty through posterior learning, dynamically updating beliefs as new data arrive. This allows it to adjust more effectively to structural

breaks, regime shifts, and volatility clustering, which are defining characteristics of gold price movements in emerging economies like India.

In contrast, the classical SSM—despite being a robust and interpretable model—relies on fixed transition and observation parameters, leading to delayed adjustment during sudden market fluctuations. Consequently, while the SSM performs reliably in stable phases, it underreacts in periods of heightened volatility. The BSTS model's probabilistic framework, by generating complete posterior predictive distributions, provides not just point forecasts but also credible intervals that enhance decision-making under uncertainty.

The study's implications extend beyond technical forecasting. For investors and traders, improved predictive precision translates into optimized hedging and portfolio allocation strategies. Policymakers and central banks can leverage Bayesian forecasts to monitor inflation expectations, currency pressures, and reserve adequacy more effectively. Additionally, the results highlight that marginal improvements in forecasting accuracy can yield substantial financial gains in high-value commodities such as gold.

Methodologically, this research reinforces that probabilistic and adaptive models outperform deterministic frameworks in environments characterized by uncertainty and data nonlinearity. It validates Bayesian structural modelling as an essential evolution in the econometric toolkit—bridging classical statistical theory with modern computational inference.

In conclusion, the study affirms that Bayesian Structural Time Series models represent a paradigm shift in volatility forecasting, combining interpretability, adaptability, and precision. Their ability to internalize uncertainty and evolve with data dynamics makes them invaluable for future research and practical application in financial and commodity market forecasting.

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